JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION AND ACTUARIAL SCIENCE

4TH YEAR 2ND SEMESTER 2018/2019 ACADEMIC YEAR

MAIN CAMPUS – INSTITUTIONAL BASED

COURSE CODE: SMA 402

COURSE TITLE: MEASURE THEORY

EXAM VENUE: **STREAM: BED AND ACT SCIENCE**

DATE: 20/08/19 **EXAM SESSION: 9.00 – 11.00AM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a) i) State three properties of Lebesgue outer measure (3mks)

ii) Given the set $E = [0, 1]$, calculate the value of $m(E')$, where E' is the set of irrational number in E. (3mks)

iii) Prove that the outer measure of a singleton set is zero. (3mks)

b) Show that for any sequence of set E_n, $m^*(\bigcup_{n=1}^{\infty} E_n) \leq \sum_{n=1}^{\infty} m^*(E_n)$ (4mks)

c) Prove that if $m^*(A) = 0$, then $m^*(A \cup B) = m^*(B)$ for any set B. (5mks)

d) i) Prove that if E is a countable set, then $m^*(E) = 0$ (5mks)

ii) Show that interval $[a, b]$ is not countable. (2mks)

e) i) Define the Lebesgue outer measure of the set $E \subseteq \mathbb{R}$ (2mks)

ii) Prove that the Lebesgue outer measure of an empty set is zero (3mks)

QUESTION TWO (20 MARKS)

a) i) State Caratheodory's measurability criteria. (2mks)

ii) Prove that if E is a countable set, then $m^*(E) = 0$ (5mks)

b) i) Describe three forms of measure (3mks)

ii) Define a property of almost everywhere in a set (2mks)

c) Suppose f and g are measurable function, prove measurability of the following:

i) $f + g$ (5mks)

ii) fg (3mks)

QUESTION THREE (20 MARKS)

a) i) Give an example of a set with outer measure zero but not countable. (1mks)

ii) Show that every interval is not countable (3mks)

iii) Show that if f is measurable function, then $\{x: f(x) = \alpha\}$ is measurable for each extended real number α. (5mks)

b) Prove that the Lebesgue outer measure is translation invariant (5mks)

c) Show that if function $h(x)$ is measurable on a measurable set E, then $|h(x)|$ is also measurable (6mks)

QUESTION FOUR (20 MARKS)

a) i) State two properties of measurable sets (2mks)

ii) Show that if $m^*(E) = 0$, then E is measurable (5mks)

b) Prove that if $f(x)$ and $g(x)$ are equivalent functions a set E and $f(x)$ is measurable, then $g(x)$ is also measurable (5mks)

c) Prove that the Dirichlet function defined by

$$f(x) = \begin{cases} 1, x \; rational \\ 0, x \; irrational \end{cases}$$

fails to have a Riemann integral over any interval $[a, b]$. Prove further that the Lebesgue intergral of $f(x)$ of any measurable set A exist and is equal to zero (8mks)

QUESTION FIVE (20 MARKS)

a) State and prove Fatous Lemma (10mks)

b) State and prove Monotone convergence theorem (10mks)