Lie Symmetry Analysis of Modified Diffusive Predator-prey Competition System of Equations

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***Abstract*– The predator-prey equations were developed and used by Lotka and Volterra to analyze the dynamics of biological systems in which two species interact, one as a predator and the other as prey [3]. Several attempts have been made in finding the exact solutions of these models using both numerical and analytical techniques [5]. Lie symmetry analysis has had applications in solving mathematical models involving non-linear differential equations; both ordinary and partial. In this paper, we have solved a modified diffusive predator-prey competition model of the form:**

$u_t - u_{xx} - u + u^2 + 2uv = 0, and, v_t - v_{xx} - \alpha v + \alpha v^2 + 3\alpha uv = 0$

; using Lie symmetry approach and obtained its general symmetry solutions. This method makes use of generator, prolongations, infinitesimal generators, symmetries and invariant solutions. The solutions obtained may be used to describe the long-term growth or decline of species in an ecosystem.

***Keywords*– Symmetries, Generators, Prolongations, Lie Group Theory, Predator-prey Equations, Invariant and Symmetry Solutions**

I. INTODUCTION

The predator-prey equations model was initially proposed by Lotka [3] in 1910 and in 1925, he utilized the equations to analyze predator-prey interactions in his book on biomathematics. The same set of equations was published in 1926 by Volterra, who had become interested in mathematical biology. Volterra proposed the classical model below to explain the oscillatory levels of fish catches in the Adriatic Sea during the years of world war I [7].

$$\frac{du}{dt} = \alpha u - \beta uv \qquad \frac{dv}{dt} = -\gamma v + \delta uv \tag{1}$$

In the above model, the functions *u (t)* and *v (t)* describe the time evolution of the numbers of prey and predators respectively; the derivatives with respect to time; *t* represents the growth rates of the two populations over time; *t* represents time; $\alpha, \beta, \gamma, \delta$ are positive real parameters describing the interactions of the two species.

Lotka proposed the same model to describe the chemical reaction which exhibit periodic behavior in the chemical concentrations. Thus, the above system (1) is known as the Lotka-Volterra model [7].

A number of modifications have been made over the years to the Lotka – Volterra equations to include several factors in the ecosystem. A natural generalization of system (1) follows if one takes into account diffusion of two species in a one-dimensional space which has led to several modifications of system (1). For instance, Kudryashov and Zakharchenko [6] investigated the analytical properties and exact travelling wave solutions of the predator-prey system below for the case $d_1 = d_2$ and (d = 1) using the method of Q-functions (the Kudryashov method) which is one of the analytical methods.

$$\frac{\partial u}{\partial t} = d_1 \frac{\partial^2 u}{\partial x^2} + u(1 - u - c_1 v), \qquad \frac{\partial v}{\partial t} = d_2 \frac{\partial^2 v}{\partial x^2} + \alpha u(1 - v - c_2 u). \tag{2}$$

In this paper, we examine a generalization of system (1) in the form below and solve it using Lie symmetry approach.

$$u_t - 2u_{xx} - u + u^2 + 2uv = 0 \qquad v_t - 2v_{xx} - \alpha v + \alpha v^2 + 3\alpha uv = 0 \tag{3}$$

Symmetry of a system of differential equations is a transformation that maps any solution to another solution of the system. In Lie's framework such transformations are groups that depend on continuous parameters and consist of point transformations (point symmetries), acting on the system's space of independent and dependent variables, or, more generally, contact transformations (contact symmetries), acting on independent and dependent variables as well as on all first derivatives of the dependent variables [4].

II. LIE SYMMETRY ANALYSIS OF THE MODEL OF DIFFUSIVE PREDATOR-PREY COMPETITION SYSTEM OF EQUATIONS

We consider a modified diffusive predator-prey competition system of equations of the form:

$$u_t - 2u_{xx} - u + u^2 + 2uv = 0$$
$$v_t - 2v_{xx} - \alpha v + \alpha v^2 + 3\alpha uv = 0 \quad (4)$$

In which the dependent variables; *u (x, t)* and *v (x, t)* represent the space and time dependent densities of the prey and predator populations respectively. The terms u_t and v_t model the rate of change for the prey and predator population with respect to time, t. The terms $2u_{xx}$ and $2v_{xx}$ model the effect of transportation in the habitat where the constant 2 represent the diffusivity of each species while propagating along the *x-axis*. The coefficient constants of *uv*; 2 and 3 are the strength of interaction for the two species.

Since our equations are of second order, we will subject them to the second prolongation of the second generator; $G^{(2)}$, which is given by [4].

$$\Pr^{(2)} G^{(2)} = \xi\frac{\partial}{\partial x} + \eta\frac{\partial}{\partial t} + \phi\frac{\partial}{\partial u} + \tau\frac{\partial}{\partial v} + \phi^x\frac{\partial}{\partial u_x} + \phi^t\frac{\partial}{\partial u_t} + \tau^x\frac{\partial}{\partial v_x} + \tau^t\frac{\partial}{\partial v_t} + \phi^{xx}\frac{\partial}{\partial u_{xx}} + \phi^{xt}\frac{\partial}{\partial u_{xt}} + \phi^{tt}\frac{\partial}{\partial u_{tt}}$$
$$+\tau^{xx}\frac{\partial}{\partial v_{xx}} + \tau^{xt}\frac{\partial}{\partial v_{xt}} + \tau^{tt}\frac{\partial}{\partial v_{tt}} \quad (5)$$

The infinitesimal criterion, which is the symmetry condition,[2], requires that;

$$\Pr^{(2)} G^{(2)}\left(u_t - 2u_{xx} - u + u^2 + 2uv\right)_{u_t - 2u_{xx} - u + u^2 + 2uv = 0} = 0$$
$$\Pr^{(2)} G^{(2)}\left(v_t - 2v_{xx} - \alpha v + \alpha v^2 + 3\alpha uv\right)_{v_t - 2v_{xx} - \alpha v + \alpha v^2 + 3\alpha uv = 0} = 0 \quad (6)$$

Which leads to;

$$\left(\xi\frac{\partial}{\partial x} + \eta\frac{\partial}{\partial t} + \phi\frac{\partial}{\partial u} + \tau\frac{\partial}{\partial v} + \phi^x\frac{\partial}{\partial u_x} + \phi^t\frac{\partial}{\partial u_t} + \tau^x\frac{\partial}{\partial v_x} + \tau^t\frac{\partial}{\partial v_t} + \phi^{xx}\frac{\partial}{\partial u_{xx}} + \phi^{xt}\frac{\partial}{\partial u_{xt}} + \phi^{tt}\frac{\partial}{\partial u_{tt}}\right.$$
$$\left.+\tau^{xx}\frac{\partial}{\partial v_{xx}} + \tau^{xt}\frac{\partial}{\partial v_{xt}} + \tau^{tt}\frac{\partial}{\partial v_{tt}}\right)\left(u_t - 2u_{xx} - u + u^2 + 2uv\right) = 0 \quad (7)$$

And;

$$\left(\xi\frac{\partial}{\partial x} + \eta\frac{\partial}{\partial t} + \phi\frac{\partial}{\partial u} + \tau\frac{\partial}{\partial v} + \phi^x\frac{\partial}{\partial u_x} + \phi^t\frac{\partial}{\partial u_t} + \tau^x\frac{\partial}{\partial v_x} + \tau^t\frac{\partial}{\partial v_t} + \phi^{xx}\frac{\partial}{\partial u_{xx}} + \phi^{xt}\frac{\partial}{\partial u_{xt}} + \phi^{tt}\frac{\partial}{\partial u_{tt}}\right.$$
$$\left.+\tau^{xx}\frac{\partial}{\partial v_{xx}} + \tau^{xt}\frac{\partial}{\partial v_{xt}} + \tau^{tt}\frac{\partial}{\partial v_{tt}}\right)\left(v_t - 2v_{xx} - \alpha v + \alpha v^2 + 3\alpha uv\right) = 0 \quad (8)$$

On differentiating equations (7) and (8) partially with respect to the partial variables $u_x, u_t, v_x, v_t, u_{xx}, v_{xx}, u_{xt}, v_{xt}, u_{tt}, v_{tt}$ and taking *u*, *v*, *t* and *x* as algebraic variables, the infinitesimal condition above reduces to;

$$\phi^t - 2\phi^{xx} - \phi + 2\phi u + 2\phi v + 2\tau u = 0$$
$$\tau^t - 2\tau^{xx} - \alpha\tau + 2\alpha\tau v + 3\alpha\phi v + 3\alpha\tau u = 0 \quad (9)$$

Which must be satisfied whenever equation (4) holds; with $\phi^t, \phi^{xx}, \tau^t, \tau^{xx}$ explicitly defined in [4].

Substituting $\phi^t, \phi^{xx}, \tau^t, \tau^{xx}$ into equation (9), we obtain the following equations;

$$\phi_t + u_t\left(\phi_u - \eta_t\right) - u_t^2\eta_u - u_x\xi_t - u_x u_t\xi_u - 2\phi_{xx} - 2\left(2\phi_{xu} - \xi_{xx}\right)u_x - 2u_x^2\left(\phi_{uu} - 2\xi_{xu}\right) - 2u_{xx}\left(\phi_u - 2\xi_x\right)$$
$$+2u_x^3\xi_{uu} + 6u_x u_{xx}\xi_u + 2u_t\eta_{xx} + 4u_t u_x\eta_{xu} + 4u_{xt}\eta_x + 2u_x^2 u_t\eta_{uu} + 2u_{xx}u_t\eta_u + 4u_{xt}u_x\eta_u - \phi + 2\phi u$$
$$+2\phi v + 2\tau u = 0 \quad (10)$$

And

$$\tau_t + v_t(\tau_v - \eta_t) - v_t^2\eta_v - v_x\xi_t - v_x v_t \xi_v - 2\tau_{xx} - 2(2\tau_{xv} - \xi_{xx})v_x - 2v_x^2(\tau_{vv} - 2\xi_{xv}) - 2v_{xx}(\tau_v - 2\xi_x)$$
$$+2v_x^3\xi_{vv} + 6v_x v_{xx}\xi_v + 2v_t\eta_{xx} + 4v_t v_x \eta_{xv} + 4v_{xt}\eta_x + 2v_x^2 v_t \eta_{vv} + 2v_{xx}v_t\eta_v + 4v_{xt}v_x\eta_v - \alpha\tau + 2\alpha\tau v \quad (11)$$
$$+3\alpha\phi v + 3\alpha\tau u = 0$$

When we replace u $_t$ by $2u_{xx} + u - u^2 - 2uv$ and v $_t$ by $2v_{xx} + \alpha v - \alpha v^2 - 3\alpha uv$ whenever they occur, we obtain a polynomial involving derivatives of *u* and *v* whose coefficients are derivatives of ξ, η, ϕ and τ as follows

$$\phi_t + (2u_{xx} + u - u^2 - 2uv)(\phi_u - \eta_t) - (2u_{xx} + u - u^2 - 2uv)^2\eta_u - u_x\xi_t - u_x(2u_{xx} + u - u^2 - 2uv)\xi_u$$
$$-2\phi_{xx} - 2(2\phi_{xu} - \xi_{xx})u_x - 2u_x^2(\phi_{uu} - 2\xi_{xu}) - 2u_{xx}(\phi_u - 2\xi_x) + 2u_x^3\xi_{uu} + 6u_x u_{xx}\xi_u + 2(2u_{xx} + u - u^2 - 2uv)\eta_{xx}$$
$$+4(2u_{xx} + u - u^2 - 2uv)u_x\eta_{xu} + 4u_{xt}\eta_x + 2u_x^2(2u_{xx} + u - u^2 - 2uv)\eta_{uu} + 2u_{xx}(2u_{xx} + u - u^2 - 2uv)\eta_u$$
$$+4u_{xt}u_x\eta_u - \phi + 2\phi u + 2\phi v + 2\tau u = 0$$

(12)

And,

$$\tau_t + (2v_{xx} + \alpha v - \alpha v^2 - 3\alpha uv)(\tau_v - \eta_t) - (2v_{xx} + \alpha v - \alpha v^2 - 3\alpha uv)^2\eta_v - v_x\xi_t - v_x(2v_{xx} + \alpha v - \alpha v^2 - 3\alpha uv)\xi_v$$
$$-2\tau_{xx} - 2(2\tau_{xv} - \xi_{xx})v_x - 2v_x^2(\tau_{vv} - 2\xi_{xv}) - 2v_{xx}(\tau_v - 2\xi_x) + 2v_x^3\xi_{vv} + 6v_x v_{xx}\xi_v + 2(2v_{xx} + \alpha v - \alpha v^2 - 3\alpha uv)\eta_{xx}$$
$$+4(2v_{xx} + \alpha v - \alpha v^2 - 3\alpha uv)v_x\eta_{xv} + 4v_{xt}\eta_x + 2v_x^2(2v_{xx} + \alpha v - \alpha v^2 - 3\alpha uv)\eta_{vv} + 2v_{xx}(2v_{xx} + \alpha v - \alpha v^2 - 3\alpha uv)\eta_v$$
$$+4v_{xt}v_x\eta_v - \alpha\tau + 2\alpha\tau v + 3\alpha\phi v + 3\alpha\tau u = 0$$

(13)

Equating to zero, the coefficients of the various monomials in the first and second order partial derivatives of u and v, we end up with determining equations for the infinitesimal transformation for the Lotka-Volterra competition system of equations which can be summarized as follows:

$$\eta_u = \eta_v = \eta_{uu} = \eta_{vv} = \eta_x = \xi_{uu} = \xi_{vv} = 0$$
$$-2\xi_u + 6\xi_u + 8\eta_{xu} = 0, and, -2\xi_v + 6\xi_v + 8\eta_{xv} = 0$$
$$2\phi_u - 2\eta_t - 2\phi_u + 4\xi_x + 4\eta_{xx} = 0, and, 2\tau_v - 2\eta_t - 2\tau_v + 4\xi_x + 4\eta_{xx} = 0$$
$$-\xi_t - 4\phi_{xu} + 2\xi_{xx} = 0, and, -\xi_t - 4\tau_{xv} + 2\xi_{xx} = 0$$
$$-\phi_{uu} + 4\xi_{xu} = 0, and, -\tau_{vv} + 4\xi_{xv} = 0$$
$$\phi_t - 2\phi_{xx} - \phi = 0, and, \tau_t - 2\tau_{xx} - \alpha\tau = 0$$

Solving the above systems of equations leads us to the solutions of the infinitesimal functions; ξ, η, ϕ and τ as,

$$\xi(x,t) = c_1 + c_6 x + 4c_5 t + 8c_4 xt$$
$$\eta(t) = c_2 + 2c_6 t + 8c_4 t^2$$
$$\phi(x,t,u) = \left(-c_4x^2 - c_5 x + \int \mu(t)dt - 4c_4 t + c_3\right)u + \alpha(x,t)$$
$$\tau(x,t,v) = \left(-c_4x^2 - c_5 x + \int \mu(t)dt - 4c_4 t + c_3\right)v + q(x,t)$$

Where $\alpha(x,t)$ and $q(x,t)$ are arbitrary solutions to the predator-prey competition system of equations (4)

Therefore, the associated infinitesimal generators are given by the following symmetries;

$$G_1 = \frac{\partial}{\partial x}, G_2 = \frac{\partial}{\partial t}, G_3 = u\frac{\partial}{\partial u} + v\frac{\partial}{\partial v}, G_4 = x\frac{\partial}{\partial x} + 2t\frac{\partial}{\partial t}$$

$$G_5 = 8t^2\frac{\partial}{\partial t} + 8xt\frac{\partial}{\partial x} - (x^2u + 4tu)\frac{\partial}{\partial u} - (x^2v + 4tv)\frac{\partial}{\partial v}$$

$$G_6 = 4t\frac{\partial}{\partial x} - xu\frac{\partial}{\partial u} - xv\frac{\partial}{\partial v}$$

Lie Groups Admitted by the predator-prey system of equations (4)

The one-parameter groups; G $_i$ admitted by the equation (4) are determined by solving the corresponding Lie equations. For instance, given the generator, $G_6 = 4t\frac{\partial}{\partial x} - xu\frac{\partial}{\partial u} - xv\frac{\partial}{\partial v}$, we have:

$$\frac{dx^*}{d\varepsilon} = 4t^*, \frac{dt^*}{d\varepsilon} = 0, \frac{du^*}{d\varepsilon} = -x^*u^*, \frac{dv^*}{d\varepsilon} = -x^*v^*$$

With initial conditions; $x^*{}_{\varepsilon=0} = x$, $t^*{}_{\varepsilon=0} = t$, $u^*{}_{\varepsilon=0} = u$ and $v^*{}_{\varepsilon=0} = v$ [1] which lead to;

$$G_1 : X(x,t,u,v;\varepsilon) \to X_1(x+\varepsilon,t,u,v)$$

$$G_2 : X(x,t,u,v;\varepsilon) \to X_2(x,t+\varepsilon,u,v)$$

$$G_3 : X(x,t,u,v;\varepsilon) \to X_3(e^{\varepsilon}x,e^{2\varepsilon}t,u,v)$$

$$G_4 : X(x,t,u,v;\varepsilon) \to X_4\left(\frac{x}{1-8\varepsilon t},\frac{t}{1-8\varepsilon t},u\sqrt{1-8\varepsilon t}.e^{\frac{-\varepsilon x^2}{1-8\varepsilon t}},v\sqrt{1-8\varepsilon t}.e^{\frac{-\varepsilon x^2}{1-8\varepsilon t}}\right)$$

$$G_5 : X(x,t,u,v;\varepsilon) \to X_5\left(x+4\varepsilon t,t,ue^{-(\varepsilon x+\varepsilon^2 t)},ve^{-(\varepsilon x+\varepsilon^2 t)}\right)$$

From the above groups, it is clear that the groups, G_1, G_2 and G_3 are merely translations and scaling, that is, trivial groups. It is only G_4, and G_5 which are non-trivial groups. Thus, genuine and therefore significant transformation groups we consider are only G_4, and G_5.

Symmetry Solutions of the Predator-prey competition system of equations (4)

The technique involved in finding the symmetry solutions is based on the fact that a symmetry group transforms any solutions of the equation under consideration into other solutions of the same equation.

Considering the symmetry group inversion theory [4], if each G_i, is a symmetry group and $u^j = \Psi^j(x, t)$ is a solution of the predator-prey equations (4), then transformation groups of equation (4), solve the equation (4). The above solution can also be written in the new variables: $\left(u^*\right)^j = \psi^j(x^*,t^*)$.

Given that x^*,t^*,u^*,v^* are group transformations of equation (4) with $\left(u^*\right)^j$, of the form $\left(u^*\right)^j = U(x,t,u,v;\varepsilon)$, for some explicit function U, and j =1,2, …, m where m is the number of dependent variables; then applying the inverse mapping, the new symmetry solution u^* satisfies the relation;

$\left(u^*\right)^j = U\left\{\left[\psi\left(g_\varepsilon^{-1}(x^*), g_\varepsilon^{-1}(t^*)\right)\right], g_\varepsilon^{-1}(x^*), g_\varepsilon^{-1}(t^*), \varepsilon^{-1}\right\}$, where ψ is any known solution of the system of equations (4) [3], If we then consider

$$G_4 : X(x,t,u,v;\varepsilon) \to X_4\left(\frac{x}{1-8\varepsilon t},\frac{t}{1-8\varepsilon t},u\sqrt{1-8\varepsilon t}.e^{\frac{-\varepsilon x^2}{1-8\varepsilon t}},v\sqrt{1-8\varepsilon t}.e^{\frac{-\varepsilon x^2}{1-8\varepsilon t}}\right)$$

and

$$G_5 : X(x,t,u,v;\varepsilon) \to X_5\left(x+4\varepsilon t,t,ue^{-(\varepsilon x+\varepsilon^2 t)},ve^{-(\varepsilon x+\varepsilon^2 t)}\right)$$

Then our new symmetry solutions are defined by:

$$u_4^*(x,t)=\psi_u\left(\frac{1}{\sqrt{1+8\varepsilon t}}\right)e^{\frac{-\varepsilon x^2}{1+8\varepsilon t}}$$

$$v_4^*(x,t)=\psi_v\left(\frac{1}{\sqrt{1+8\varepsilon t}}\right)e^{\frac{-\varepsilon x^2}{1+8\varepsilon t}}$$

$$u_5^*(x,t)=\psi_u e^{-\varepsilon x+\varepsilon^2 t}$$

$$v_5^*(x,t)=\psi_v e^{-\varepsilon x+\varepsilon^2 t}$$

Where $u=\psi_i$ is any known (invariant) solution of the system of equations (4).

Given the following invariant solutions calculated for the generators G_1, and G_2, $G_1=\frac{\partial}{\partial x}$ and $G_1=\frac{\partial}{\partial t}$

$$\varphi(t)=k_1e^t+\frac{1}{t+k_2}$$

$$\sigma(t)=k_4e^{\alpha t}+\frac{1}{\alpha t+k_5}$$

$$g(x)=-\frac{1}{2}\ln\left(x+c_1\right)-\frac{1}{2}c_2e^x$$

$$\mu(x)=-\frac{\alpha}{2}\ln\left(\alpha x+c_4\right)-\frac{1}{\alpha}c_5e^{\alpha x}$$

We obtain the following list of new symmetry solutions; $u^j(x,t)$ of the predator-prey competition system of equations (4)

$$u_4^1(x,t)=\left(k_1e^t+\frac{1}{t+k_2}\right)\frac{1}{\sqrt{1+8\varepsilon t}}e^{\frac{-\varepsilon x^2}{1+8\varepsilon t}}$$

$$v_4^1(x,t)=\left(k_4e^{\alpha t}+\frac{1}{\alpha t+k_5}\right)\frac{1}{\sqrt{1+8\varepsilon t}}e^{\frac{-\varepsilon x^2}{1+8\varepsilon t}}$$

$$u_4^2(x,t)=\left(-\frac{1}{2}\ln\left(x+c_1\right)-\frac{1}{2}c_2e^x\right)\frac{1}{\sqrt{1+8\varepsilon t}}e^{\frac{-\varepsilon x^2}{1+8\varepsilon t}}$$

$$v_4^2(x,t)=\left(-\frac{\alpha}{2}\ln\left(\alpha x+c_4\right)-\frac{1}{\alpha}c_5e^{\alpha x}\right)\frac{1}{\sqrt{1+8\varepsilon t}}e^{\frac{-\varepsilon x^2}{1+8\varepsilon t}}$$

$$u_5^3(x,t)=\left(k_1e^t+\frac{1}{t+k_2}\right)e^{-\varepsilon x+\varepsilon^2 t}$$

$$v_5^3(x,t)=\left(k_4e^{\alpha t}+\frac{1}{\alpha t+k_5}\right)e^{-\varepsilon x+\varepsilon^2 t}$$

$$u_5^4(x,t)=\left(-\frac{1}{2}\ln\left(x+c_1\right)-\frac{1}{2}c_2e^x\right)e^{-\varepsilon x+\varepsilon^2 t}$$

$$v_5^4(x,t)=\left(-\frac{\alpha}{2}\ln\left(\alpha x+c_4\right)-\frac{1}{\alpha}c_5e^{\alpha x}\right)e^{-\varepsilon x+\varepsilon^2 t}$$

Where k_i , α and c_i are arbitrary constants.

III. CONCLUSIONS

In this paper, we have managed to use Lie symmetry approach to prolong the infinitesimal generator of our system (4), which enabled us to obtain its determining equations. After solving the determining equations, we obtained the infinitesimal generators which we used to construct the Lie groups admitted by the system (4) and obtained the group transformations of solutions of the predator-prey equations (4). Finally, we found the invariant solutions which enabled us to develop new symmetry solutions of the predator-prey competition system of equations (4). The solutions obtained maybe used to predict the growth or decline of species in an ecosystem and can also satisfy the typical requirements occurring in biologically motivated problems describing the interaction of prey–predator type between two species.

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