JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

SPECIAL RESITS EXAMINATIONS, NOVEMBER 2020

COURSE CODE: SMA 200

COURSE TITLE: Calculus Ii

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (COMPULSORY) (30 marks)

a) State the fundamental theorem of calculus (2 marks)

b) Using $\int_0^{10} \frac{1}{5} x dx$, show that the idea behind integration is that we can effectively compute many quantities by breaking them into small pieces, and then summing the contributions from each small part (4 marks)

c) Verify by differentiation that the formula is correct:
$\int \cos x dx = \sin x + c$ (5 marks)

d) Using appropriate substitution, evaluate the indefinite integral
$\int \frac{16x}{\sqrt{8x^2+1}} dx$ (5 marks)

e) Evaluate the given integral using the given substitution to reduce to standard form:
$\int_1^2 2(2x+4)^5 dx$, $u = 2x+4$ (4 marks)

f) Find the length of the curve $y = \frac{2\sqrt{2}}{3} x^{3/2} - 1$ from $x = 0$ to $x = 1$ (4 marks)

g) Evaluate appropriately $\int_0^1 e^{x^2} dx$ using Simpson's rule with eleven ordinates. (6marks)

QUESTION TWO (20 marks)

a) Evaluate the following integral by completing the square and using a substitution to reduce it to standard form:
$\int_1^2 \frac{8}{x^2 - 2x + 2} dx$ (6 marks)

b) Evaluate the following integral by separating the fraction and using a substitution (if necessary) to reduce it to standard form:
$\int_0^{\frac{1}{2}} \frac{2-8x}{1+4x^2} dx$ (4 marks)

c) By making the appropriate substitution for u in the integral below:
   (i) Express the integral in terms of u. (3 marks)
   (ii) Evaluate the integral as function of x. (3 marks)

$$\int_1^3 \frac{x+3}{(x-2)^2} dx$$

d) Evaluate the following integral by multiplying by a form of one and using a substitution (if necessary) to reduce it to standard form:
$\int_0^{\pi/4} \frac{1}{1+\cos\theta} d\theta$ (5 marks)

QUESTION THREE (20 marks)

a) Determine the value of the integral

$\int \cos^5 \theta d\theta$, (Hint: $\cos^2 \theta = 1 - \sin^2 \theta$) (5 marks)

b) Perform long division on the integrand, write the proper fraction as a sum of partial fractions, and then evaluate the integral

$\int \frac{x^3 + 4x^2 - x}{(x+2)(x+1)} dx$ (8 marks)

a) Integrate by parts

$\int x^3 e^x dx \quad dx$ (7 marks)

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QUESTION FOUR (20 marks)

a) Find the volume of the solid generated by revolving the region bounded by the curve $y = 4 - x^2$ and line $y = 2 - x$ about the x-axis. (7 marks)

b) Determine the area of the surface generated by revolving the curve $y = \sqrt{2x - x^2}$, $0.5 \leq x \leq 1.5$ about the x-axis. (6 marks)

c) Find the area of the region enclosed by the line $x + y^2 = 3$ and the curve $4x + y^2 = 0$. (7 marks)

QUESTION FIVE (20 marks)

a) Evaluate $\int_0^1 e^{x^2} dx$ by Simpson's rule taking ten intervals (5 marks)

b) Find a power series for the logarithmic function

$L(x) = \ln(1+x)^3$ (6 marks)

c) Show that the Taylor series about $x = 0$ for the function $f(x) = \cos x$ is $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$

(5 marks)

d) Evaluate the following integral

$\int \frac{x^2 \tan^{-1} x^3}{1 + x^6} dx$ (4 marks)