JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR THE DEGREE IN BSC. ACTUAL SCIENCE, BED SCI./ ARTS
SPECIAL RESIT 2020/2021 ACADEMIC YEAR

COURSE CODE: **SMA 300**
COURSE TITLE: **Real Analysis**
EXAM VENUE:
STREAM: **BSC. ACTUARIAL, BED SCI/ARTS**

DATE:…………… **EXAM SESSION …………………………….**

TIME: **2 HOURS**

Instructions:

1. **Answer all questions in Section A and any other 2 questions in Section B**
2. **Candidates are advised not to write on the question paper**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room**

SECTION A

QUESTION ONE compulsory (30 MARKS)

a) Define the following terms: (6 marks)
   (i) Metric space
   (ii) Riemann integrable function
   (iii) Sequence

b) Find a simple expression for nth terms of each sequence (4 marks)
   i) $1, \frac{1}{2}, \frac{1}{3}, \ldots$
   ii) $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \ldots$

c) Show that $\lim_{n\to\infty}\left\{\frac{2n+1}{n+1}\right\} = 2$ (5 marks)

d) Let (x_n) and (y_n) be two converging sequences as $n \to \infty$ with the limits l and h respectively. Show that $x_n + y_n \to l + h$.
   (7 marks)

e) Show that $d(\Re, d)$ is a metric space whenever $d(x, y) = |x - y|, x, y \in \Re$ (8 marks)

SECTION B

QUESTION TWO (20 MARKS)

a) Suppose $f : [a, b] \to \Re$ is bounded. Prove that $\int_{-a}^{b} f \le \int_{a}^{-b} f$. (6 marks)

b) If f is a Riamann integrable function on [a,b]. Show that $U(f.P) - L(f, P) < \varepsilon$ (10 marks)

c) Using Bolzano-Wierstrass theorem show that the bounded sequence below is convergent.
   $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \ldots$ (4 marks)

QUESTION THREE (20 MARKS)

a) List the first three terms of the sequence (2 marks)
   (i) $(-1)^n n$
   (ii) $\frac{n+1}{n^2}$

b) Prove that sequence a_n has at-most one limit. (8 marks)

c) State and prove monotone convergence theorem (10 marks)

QUESTION FOUR (20 MARKS)

a) Distinguish between limit inferior and limit superior. (4 marks)

b) Show that every Cauchy sequence is bounded. (8 marks)

c) Prove that the sequence $\left(\frac{1}{2^n}\right)$ is Cauchy sequence. (8 marks)

QUESTION FIVE (20 MARKS)

a) Show that the series $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$ converges to 1 (4 marks)

b) State the following theorems and Lemma (8 marks)
   i) Comparison test theorem
   ii) Integral test theorem
   iii) Bolzano-Weirstrass theorem
   iv) Squeeze Lemma

c) Prove that the set of real numbers in the interval (0,1) is uncountable. (8 marks)