JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

FIRST YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE -DECEMBER 2020

SMA *3121*: Mathematics II (Special Exam)

INSTRUCTIONS:

1. This examination paper contains five questions. Answer **question one,** and **any other two** questions.
2. Start each question on a fresh page.
3. Indicate question number clearly at the top of each page.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Given two matrices $A=\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$ and $B=\begin{bmatrix} 3 & -2 \\ 1 & 2 \end{bmatrix}$. Find

i. 2A-3B (2 marks)

ii. BA (2 marks)

iii. B^{-1} (3 marks)

b) Given two points P(0, -1) and Q(4, 1). Find the equation of the line that is perpendicular to PQ and passes through the midpoint of PQ. (4 marks)

c) Evaluate

i) $\lim_{x\to 1}(x^2+1)$ (2 marks)

ii) $\lim_{x\to 3}(x^2+x+6)$ (3 marks)

d) Determine the area between the curve $y=x^3-2x^2-8x$ and x-axis. (5 marks)

e) Find ${dy}/{dx}$ in $x^2-y^2=1$. (3 marks)

f) Consider the three points A(-2,1) B(2,3) and C(3,1).

i) Find the length of each side of the triangle. (3 marks)

ii) Verify that the triangle is right angle triangle (2 marks)

iii) Find the area of the triangle. (1 mark)

QUESTION TWO (20 MARKS)

a) Given the matrix $A=\begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 4 & 2 & 1 \end{bmatrix}$. Find

i) $|3A|$ (2 marks)

ii) The adjoint of A. (4 marks)

iii) Inverse of A. (3 marks)

b) Solve the system of equations using Cramers rule (6 marks)

$$x_1 + 3x_2 + x_3 = -2$$
$$2x_1 + 5x_2 + x_3 = -5$$
$$x_1 + 2x_2 + 3x_3 = 6$$

c) Evaluate $\int 3te^{2t} dt$ (5 marks)

QUESTION THREE (20 MARKS)

a) Find the derivative of the polynomial

i) $y = x^3 + \frac{4}{3}x^2 - 5x + 1$ (3 marks)

ii) $y = \frac{x^2 - 1}{x^3 + 1}$ (3 marks)

b) Determine if the following functions are continuous or discontinuous.

i) $f(x) = \frac{3x^2 - 7x + 2}{x - 2}$ (3 marks)

ii) $f(x) = \frac{1}{x^2 + 1}$ (3 marks)

c) The concentration C in mg of a chemical in bloodstream t hours after injection into the muscle tissue can be modeled by $C = \frac{3t}{27 + t^3}; t \geq 0$. Determine the time when the concentration reaches its highest level. (5 marks)

d) Find the distance between A(1,1) and B(3,4). (3 marks)

QUESTION FOUR (20 MARKS)

a) Use Gauss-Jordan elimination to solve (6 marks)

$3x - y = 7$

$2x + 5y = 16$

b) Find dy/dx if $2x^3 - 3y^2 = 8$ (6 marks)

c) Find the slope m and y-intercept of the equation 2x+4y=8. (3 marks)

d) Solve the following equation for the variable x $\begin{vmatrix} x & x+1 \\ -1 & x-2 \end{vmatrix} = 7$. (5 marks)

QUESTION FIVE (20 MARKS)

a) Evaluate the given definite integral (5 marks)

$$\int_{-1}^{0} (-3x^5 - 3x^2 + 2x + 5)dx$$

b) Given a system of equations

$2x_1 + 7x_2 + 3x_3 = 7$

$x_1 + 2x_2 + x_3 = 2$

$x_1 + 5x_2 + 2x_3 = 5$

(i) Express the system in the form of matrix equation $AB = C$, where A is a 3×3 matrix of coefficients of the variables, B and C are suitable column matrices. (2 marks)

(ii) Determine the adjoint of the matrix A. (5 marks)

(iii) Hence solve the system of equations. (4 marks)

c) Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangent? If so where? (4 marks)