JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION AND ACTUARIAL SCIENCE

SPECIAL RESIT

MAIN CAMPUS

COURSE CODE: SMA 300

COURSE TITLE: Real Analysis

EXAM VENUE: **STREAM: Bed And Act Science**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

<u>Instructions:</u>

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a) i) Find a simple expression for the n^{th} term of the following sequence

$1, \frac{1}{2}, \frac{1}{3}, \ldots$ (2marks)

ii) Differentiate between a bounded sequence and a convergence sequence (4marks)

iii) Determine whether the sequence $\frac{1}{n}$ is bounded (2marks)

iv) Show that $\lim_{n\to\infty}(\frac{3n+2}{n+1}) = 3$ (5marks)

b) State and prove Bolzano-Weirstrass theorem (5marks)

c) Suppose that $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ are convergent series. Then prove that

i) $\sum_{k=1}^{\infty}(a_k + b_k) = \alpha + \beta$ (4marks)

ii) $\sum_{k=1}^{\infty} ca_k = c\alpha$ (3marks)

d) i) Define a Reimann integrable function (2marks)

ii) Prove that the function $f: [0,1] \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} 1, if\ x \in \mathbb{Q} \\ 0, if\ x \notin \mathbb{Q} \end{cases}$$

is not Riemann integrable (3marks)

QUESTION TWO(20 MARKS)

a) i) Define a Cauchy sequence (2marks)

ii) Show that every Cauchy sequence converges (4marks)

b) i) Prove that if (x_n) is a convergent sequence then its limit is unique (5marks)

ii) Show that if (x_n) is a sequence that converges to l then every subsequence (x_{nr}) converges to l (4marks)

c) Let (x_n) and (y_n) be two sequences converging to l and h respectively as $n \to \infty$. Then show that $x_n + y_n \to l + h$ (5marks)

QUESTION THREE (20 MARKS)

a) i) Define a metric space (4marks)

ii) Show that $(\mathbb{R}, d)$ is a metric space where $d(x, y) = |x - y|$ (8marks)

b) Prove that the set $C_R[0,1]$ of all real-valued continuous functions defined on $[0,1]$ is a metric space with respect to the metric defined as

$d(f, g) = Sup\{|f(x) - g(x)|: x \in [0,1]\}$ where $f, g \in C_R[0,1]$ (8marks)

QUESTION FOUR (20 MARKS)

a) i) What is a monotone sequence (2marks)

ii) Show that if (x_n) is monotonically increasing and is bounded above, then x_n converges to its Supremum (5marks)

b) i) Define a divergent sequence (2marks)

ii) Show that the sequence $(-1)^n$ is divergent although it is bounded by 1 (4marks)

c) Prove that a sequence (x_n) is bounded if and only if there is some $K \geq 0$ such that $|x_n| \leq K$ for all n (7marks)

QUESTION FIVE (20 MARKS)

a) Let (a_n) be a sequence of positive real numbers and suppose that there is some positive function ψ such that the sequence of integrals $\left(\int_1^n \psi(x)dx\right)_{n\in\mathbb{N}}$ converges as $n \to \infty$ and such that, for each $k \geq 2$, $a_k \leq \psi(x)$ for all $(k-1) \leq x \leq k$. Show that $\sum_{k=1}^{\infty} a_k$ is convergent (10marks)

b) Suppose that $f: [a, b] \to \mathbb{R}$ is bounded. Then f is integrable on $[a, b]$ if f for every $\in > 0$there exist a partition P of $[a, b]$ such that $U(f, P) - L(f, P) < \in$ (10marks)