JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

2^{ND} YEAR 2^{ND} SEMESTER 2018/2019 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: SAC 202

COURSE TITLE: LIFE TESTING ANALYSIS

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE

(a) Define the following words as used in Survival Analysis

i. Time - to- event.

ii. Distribution Function.

iii. Cumulative Hazard.

iv. Censoring.

v. Truncation. [5 marks]

(b) Consider an exponential density function with parameter $\mu = 0.4$. Determine the following;

i. Cumulative function. [2 marks]

ii. Survival probability at time 10 weeks. [2 marks]

iii. Hazard function at time 2 weeks. [2 marks]

(c) The complete life expectation of a life age x, is $\mathring{e}_x$. Show that

$$\mathring{e}_x = \int_0^\infty {}_tp_x dt.$$

[5 marks]

(d) If $\mu_x = 0.010908 + 0.001(x - 70)$, for $x \geq 55$. Calculate ${}_5q_{60}$. [4 marks]

(e) Suppose that Gompertz Law applies with $\mu_{30} = 0.00013$ and $\mu_{50} = 0.000344$, calculate the probability that (40) will survive for 10 years.

[4 marks]

(f) Most actuaries use the exponential distribution of the form

$$s(t) = 1 - e^{-\lambda t}$$

as a premise for follow up studies. Derive the distribution function and the hazard function, and comment on the structure of the cumulative hazard function.

[6 marks]

QUESTION TWO

(a) Sketch the following as functions of age x for a typical (human) population and comment on the major features.

i. μ_x, [2 marks]

ii. l_x, and [2 marks]

iii. d_x [2 marks]

(b) Calculate the survival function and the pdf for T_x using Gompertz law of mortality with $B = 0.0003$, $c = 1.07$ for $x = 20$, $x = 50$, $x = 80$. Plot the results and comment on the features of the graphs. [10 marks]

(c) Given that $e_{50} = 30$ and $\mu_{50+t} = 0.005$, for $0 \leq t \leq 1$, what is the value of e_{51}? [4 marks]

QUESTION THREE

(a) The population of elderly people in a prison is observed during the period 1 January 1994 to 31 December 1996. The duration of residence (measured to the nearest number of months) is recorded for those who die during the period, for those who are released from the prison during the period and for those who are still in residence on 31 December 1996. The recorded data measured in months are

6^{+} 6 6 6 7 9^{+} 10^{+} 10 11^{+} 13 16 17^{+} 20 23^{+}

where + indicates those who were released from the prison during the period or who were still in residence on 31 December 1996.

i. State the type(s) of censoring inherent in these data. [2 marks]

ii. Determine the product-limit (Kaplan-Meier) estimate of the survival function, $S(t)$, where t is the duration of residence in the prison. [7 marks]

iii. State the assumptions underlying the estimate in (ii), and explain how each of these assumptions would apply to these data. [4 marks]

(b) An investigation is carried out into the lifestyle of male accountants. A group of 10000 accountants is selected at random on 1 January 2001. Each member of the group supplies detailed personal information as at 1 January 2001 including name, address date of birth and marital status. The same information is collected as at each 1 January in the years 2002, 2003, 2004 and 2005. The investigation closes in 2005. A PhD student wishes to use the data from this investigation for her thesis on the mortality of married men. Describe the ways in which the available data for this investigation are censored. [4 marks]

(c) A trial for the life of a new battery was carried out by placing 1,000 batteries inside 1,000 fluffy electrical toys. The toys were turned on and left to run for 24 hours. A researcher returned every hour to count how many toys had stopped operating. On close analysis it was discovered that some of the toys had ceased to operate due to mechanical problems and that some of the toys were not switched on in the first place. Some of the toys were still running after 24 hours. For each of the following types of censoring, state with reasons whether it is present in the investigation:

i. left-censoring
ii. interval censoring
iii. type I censoring
iv. non-informative censoring [4 marks]

QUESTION FOUR

(a) Consider the following distribution function of the future lifetime T of a newborn.

$$F_0(t) = 1 - (1 - \frac{t}{120})^{\frac{1}{6}}, \ 0\, t \leq 120.$$

Use it to find

i. a probability that a newborn survives beyond age 30, [2 marks]

ii. the probability that a life age 30 dies before age 50, [2 marks]

iii. the probability that a life age 40 survives beyond age 65. [2 marks]

(b) If $\hat{\mu}_{60} = 0.01$, $\hat{\mu}_{61} = 0.02$, $\hat{\mu}_{62} = 0.03$, estimate the values of p_{60}, ${}_2p_{60}$ and ${}_3p_{60}$. [3 marks]

(c) Give reasons as to why the Cox PH model is commonly used. [3 marks]

(d) Consider a discrete random variable T.

i. Show that $S(t) = \prod_{t_j<t}(1 - \lambda_j)$. [3 marks]

ii. Derive the Kaplan Meier estimator of the survival function. [3 marks]

QUESTION FIVE

(a) In a certain population, the force of mortality equals 0.025 at all ages. Calculate:

i. the probability that a life age 10 will die before age 12, [2 marks]

ii. the probability that a life age 5 will die between ages 10 and 12 [2 marks]

iii. the complete expectation of life of a new-born baby [2 marks]

iv. the curtate expectation of life of a new-born baby. [2 marks]

(b) 12 brain tumor patients were randomized into radiation or radiation+chemotherapy. One year after the start of the study, survival time in weeks ere recorded as follows;

Group 0 RT	10	26	28	30	41	12^+
Group 1 RT+CT	24	30	42	40^+	30^+	42^+

Apply a logrank test at $\alpha = 0.05$. [6 marks]

(c) Show that if the hazard function has the form $\alpha\beta(\alpha\, t)^{\beta-1}\exp[(\alpha\, t)^{\beta}]$, then the survival function is $\exp\{-(\exp(\alpha\, t)^{\beta} - 1)\}$. [6 marks]