JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

YEAR FOUR SEMESTER ONE EXAMINATION

SMA 405 : PARTIAL DIFFERENTIAL EQUATION I(Special Resit)

INSTRUCTION: Answer Question ONE and ANY other TWO questions.

QUESTION ONE (COMPULSORY)

a) State the order and degree of the partial differential equations below

i) $\frac{\partial^2 y}{\partial x^2} + \left(\frac{\partial y}{\partial x}\right)^3 + \left(\frac{\partial^3 z}{\partial x^3}\right)^4 = 0$

ii) $\left(\frac{\partial y}{\partial x}\right)^4 + \frac{\partial^2 z}{\partial x^2} + \frac{\partial^3 z}{\partial x^3} = 0$ (4 marks)

b) Define the following

i) Total differential equation

ii) Non-Linear partial differential Equation

iii) Semi-linear partial differential Equation

iv) Quasi-linear partial differential equation (8 marks)

c) Solve the simultaneous Differential equation

$$\frac{dx}{xz(z^2+xy)} = \frac{dy}{-yz(z^2+xy)} = \frac{dz}{x^4}$$ (6 marks)

d) Find the orthogonal trajectory on the cone $x^2 + y^2 = z^2 \tan^2\alpha$ of its intersection with with the family of planes parallel to $z = 0$ (8 marks)

e) Solve the following differential equations by inspection

i) $df(x,y) = \frac{xdy + ydx}{x^2}$

ii) $df(x,y) = \frac{xdy + ydx}{x^2 + y^2}$ (4 marks)

QUESTION TWO (20 marks)

a) By eliminating the arbitrary constants *a* and *b* from $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ form a partial differential equation (4 marks)

b) Solve the homogeneous equation

$(x^2y - y^3 - y^2z)dx + (xy^2 - x^2z - x^3)dy + (xy^2 + x^2y)dz = 0$ (10 marks)

c) By choosing appropriate multipliers solve

$$\frac{dx}{4y-3z}=\frac{dy}{4x-2z}=\frac{dz}{2y-3x}$$ (6 marks)

QUESTION THREE (20 marks)

a) Solve the Pfaffian differential equation

$$(yz+xyz)dx+(zx+xyz)dy+(xy+xyz)dz=0$$ (5 marks)

b) Find $f(y)$ such that the Pfaffian differential equation

$\{(yz+z)/x\}dx-zdy+f(y)dz=0$ is integrable hence solve it. (10 marks)

c) Use Lagrange's method to solve $xyp+y^2q=zxy-2x^2$ (5 marks)

QUESTION FOUR (20 marks)

a) Show that the equation $xp-yq=x$ and $x^2p+q=xy$ are compatible hence find their solution. (10 marks)

b) Solve $(x^2+y^2)p+2xyq=z(x+y)$ (5 marks)

c) Form a partial differential equation by eliminating the arbitrary function *f* from the function $x+y+z=f(x^2+y^2+z^2)$ (5 marks)

QUESTION FIVE (20 marks)

a) Solve the Cauchy's problem for $zp+q=1$ where the initial data curve is $x_0=\mu, y_0=\mu, z_0=\frac{\mu}{2}$ for $0\le\mu\le1$ (8 marks)

b) Use Charpit's method to find the complete integral of $p^2-y^2q=y^2-x^2$ (12 marks)