JARAMOGI OGINGA ODINGA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE /ARTS

MAIN REGULAR

COURSE CODE: SMA 403

COURSE TITLE: TOPOLOGY

EXAM VENUE **STREAM: BED SCI/ARTS**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

INSTRUCTIONS:

1. This examination paper contains five questions. Answer **question one,** and **any other two** questions.
2. Start each question on a fresh page.
3. Indicate question number clearly at the top of each page

SECTION A

QUESTION ONE

a) Define the following with respect to a metric space:
   (i) Neighbourhood (3 marks)
   (ii) Interior point (3 marks)

b) Show that the intersection of a finite family of open sets is itself open in a metric space. (6 marks)

c) Define a topology and suppose that $X = \{a, b, c, d, e, f\}$ and $\tau = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e, f\}\}$, Is τ a topology on X? (5 marks)

d) Let (X,τ) be a topological space and$A \subseteq X$. If $X = \{1,2,3,4\}$, $A = \{1,2\}$, $B = \{3,4\}$,

$$\tau = \{\phi,\ \{1,2\}, \{1,2,3\}, \{1,2,4\}, X\}.$$

Find

(i). Int(A) (1 mark)

(ii). Ext(A) (2 marks)

(iii). Bdy(A) (2 marks)

(iv). Bdy(B) (5 marks)

e) Show that the indiscrete topological space is indeed a topological space. (3 marks)

SECTION B

QUESTION TWO

a) Let (X,τ) be a topological spaces. Show that $\tau_1 \cap \tau_2$ is a topology on X. (6 marks)

b) Show that the taxicab metric d for $\mathbb{R}^n$ indeed a metric space. (6 marks)

c) Let $X = \{1,2,3,4,5\}$ $\tau = \{X, \phi, \{5\},\ \{2,3\}, \{2,3,5\}, \{1,2,3,4\}\}$ and $A = \{1,4,5\}$. Find all the limit points of A. (6 marks)

d) Define the closure of a set. (2 marks)

QUESTION THREE

a) Consider (X,τ) be a topological spaces and $A \subseteq X$. Define$\tau_A = \{A \cap u | u \in \tau\}$. Show that τ_A is a topology on A. (6 marks)

b) Prove that a topological space (X,τ) is a T_1-space if and only if every singleton subset of X is closed. (6 marks)

c) $X = \{1,2,3\}$, $\tau_1 = \{X, \emptyset, \{1\}, \{1,2\},\ \{1,3\}\}$ $\tau_2 = \{X, \emptyset,\ \{2\}\}$

(i) Is $\tau_1 \cup \tau_2$ a topology on X. (5 marks)

(ii) Which one of the topologies is coarser (finer)? Explain you answer? (3 marks)

QUESTION FOUR

a) Given $X = \{1, 2, 3, 4, 5\}$ $\tau = \{X, \emptyset, \{5\}, \{2,3\},\ \{2,3,5\},\ \{1,2,3,4\}\}$ $A = \{5\}$, $B = \{3, 4\}$

Find (i). $\overline{A}$ (2 marks)

(ii). Boundary of B (4 marks)

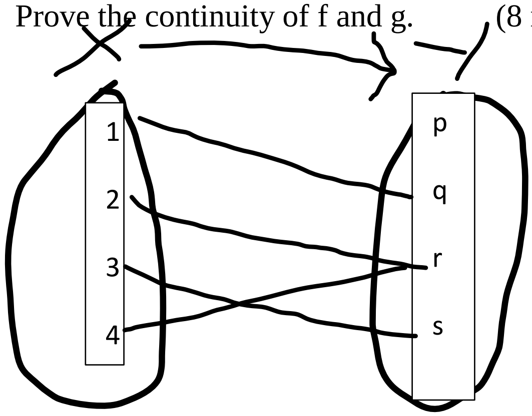
(iii) Limit points of A. (4 marks)

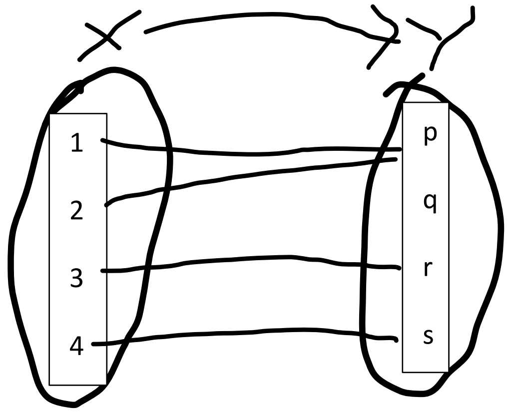
b) Let (X,τ) be a topological space and $A \subseteq X$. Prove that A is closed if and only if it contains all its limit points. (10 marks)

QUESTION FIVE

a) et $X = \{a, b, c\}$ and $\mathfrak{B} = \{\{a\}, \{c\}, \{a, b\}, \{b, c\}\}$. Show that $\mathcal{B}$ is not a basis for any topology on X? (6 marks)

b) Let $X = \{1,2,3,4\}$ and $Y = \{p, q, r, s\}$ $\tau_X = \{\emptyset, \{1\}, \{1,2\}, \{1,2,3\}, X\}$ $\tau_Y = \{\emptyset, \{p\}, \{q\}, \{p, q\}, \{q, r, s\}, Y\}$. Let f and g be two functions defined by the diagrams. Prove the continuity of f and g. (8 marks)





c) State the following separation axioms

i) T_0-space (2 marks)

ii) Haursdorff space (2 marks)

iii) Regular space (2 marks)