JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (SCIENCE)

2ND YEAR 2ND SEMESTER 2016/2017 ACADEMIC YEAR

MAIN

REGULAR

COURSE CODE: SPH 205

COURSE TITLE: Mathematical Methods For Physics Ii

EXAM VENUE: **STREAM: EDUCATION**

DATE: **EXAM SESSION:**

TIME: 2:00 HRS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

SECTION A

QUESTION 1(30 MARKS)

a) (i) Given that $z = x^2y^2 + 3xy$, determine $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ **(2 marks)**

(ii) Write an equation for the total differential of the function $f = f(x_1, x_2, \dots x_n)$ **(1 mark)**

b) Evaluate $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^{10}$ leaving your answer in the form $a + ib$. **(3 marks)**

c) Show that the complex-valued function $f(z) = z^2$ is analytic. **(2 marks)**

d) Define the following terms.

(i) Power series **(1 mark)**

(ii) Essential singularity **(2 marks)**

e) State the fundamental theorem of algebra. **(1 mark)**

f) (i)Find the determinant of $A = \begin{pmatrix} 3 & 2 & 0 & 1 \\ 4 & 0 & 1 & 2 \\ 3 & 0 & 2 & 1 \\ 9 & 2 & 3 & 1 \end{pmatrix}$ **(4 marks)**

(ii) Given the system of linear equations

$a_1x_1 + a_2x_2 = a_3$

$b_1x_1 + b_2x_2 = b_3$, obtain an expression for x_2 in form of a ratio of two determinants. **(3 marks)**

g) Distinguish between a Hilbert space and the Gram-Schmidt orthogonalization. **(2 marks)**

h) Show that in a Fourier series of a function $f(x)$ in the interval $[-P\ \ P]$, the Fourier coefficient a_0 is given by

$$a_0 = \frac{1}{P}\int_{-P}^{P} f(x)dx$$ **(4 marks)**

i) (i) Distinguish between a first order linear and homogeneous ordinary differential equation. **(2 marks)**

(ii) Solve the equation $\frac{dy}{dx} + y = e^x$ **(3 marks)**

SECTION B

Answer any TWO questions in this section

QUESTION 2(20 MARKS)

a) Determine the partial second derivatives of $f(x, y) = e^{2x}\cos(y - x)$, hence show that the second partial derivative is a commutative operation. **(9 marks)**

b) Derive the De Moivre's theorem. **(5 marks)**

c) Calculate the cube root of 8. **(6 marks)**

QUESTION 3 (20 MARKS)

a) Derive the Cauchy Riemann Equations from first principles. **(9 marks)**

b) Expand $\frac{1}{(Z+1)(Z-3i)}$ in a Laurent series about the point $Z = -1$ **(7 marks)**

c) Show that the set $S = \left\{\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)\right\}$ is orthonormal. **(4 marks)**

QUESTION 4 (20 MARKS)

a) (i) Name an eigen value equation in quantum mechanics. **(1 mark)**

(ii) Let $B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$. Find the eigen values of B and the associated eigenvectors. **(12 marks)**

b) Show that the differential equation

$$(x^2 + y^2)dx + 2xydy = 0 \text{ is exact,}$$

hence solve it. **(7 marks)**

QUESTION 5 (20 MARKS)

a) Diagonalize the matrix $\begin{pmatrix} p & -q \\ q & p \end{pmatrix}$ where p and q are real numbers and $q \neq 0$

(14 marks)

b) Define a Fourier series of a function $f(x)$. **(2 marks)**

c) Given the box function which can represent a single pulse,

$f(x) = \begin{cases} 1, & -a \leq x \leq a \\ 0, & x > a \end{cases}$, find the Fourier transform of $f(x)$ **(4 marks)**