JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES
UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (SCIENCE)
4^{TH} YEAR 2^{ND} SEMESTER 2017/2018 ACADEMIC YEAR
MAIN REGULAR

COURSE CODE: SPH 403

COURSE TITLE: Quantum Mechanics Ii

EXAM VENUE: **STREAM: EDUCATION**

DATE: **EXAM SESSION:**

TIME: 2:00 HRS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Attempt question ***1(Compulsory)*** and any other ***TWO*** questions in **section B.**

SECTION A

QUESTION 1 (30 MARKS)

a)

i. State any **TWO** postulates of Quantum mechanics **(2 marks)**

ii. Show that if two wave functions ψ_1 and ψ_2 satisfy the time-dependent Schroedinger equation, then their superposition $\alpha\psi_1 + \beta\psi_2$ in which α and β are constants is also a wave function satisfying the same Schroedinger equation.

(3 marks)

b) Write down the TWO main mathematical properties of the quantum wave function that formed the basis for Dirac's formulation of Quantum mechanics. **(2 marks)**

c) Distinguish between spin-orbit coupling and Schroedinger picture. **(2 marks)**

d) Determine the THREE components of orbital angular momentum in cartesian coordinate system. **(3 marks)**

e) Calculate the commutation relation $\left[\hat{L}_x, \hat{L}_y^2\right]$ **(4 marks)**

f) Derive the energy spectrum of a quantized linear harmonic oscillator in a number state $|n\rangle$

given the Hamiltonian $\hat{H} = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)$ where the symbols have their usual meaning

(4 marks)

g) Distinguish between time-independent perturbation theory and time-dependent perturbation theory. **(2 marks)**

h) State the selection rules for allowed transitions in hydrogen atom. **(2 marks)**

i) Write down the Classical and corresponding Quantum mechanical form of the Hamiltonian of a two particle system **(2 marks)**

j) List the state vectors $|u\rangle;|d\rangle$ and use them to determine the spin state transition operators of a two-state system, $\hat{S}_+$; S_- **(4 marks)**

SECTION B

Attempt any TWO questions in this section.

QUESTION 2 (20 MARKS)

a) Determine the expression for the ladder operator $\hat{L}_+$ of the angular momentum given

$\hat{L}_x = i\hbar\left(\sin\phi\frac{\partial}{\partial\theta} + \cos\phi\cot\theta\frac{\partial}{\partial\phi}\right)$ and $\hat{L}_y = i\hbar\left(-\cos\phi\frac{\partial}{\partial\theta} + \sin\phi\cot\theta\frac{\partial}{\partial\phi}\right)$ **(5 marks)**

b) The number state vector $|n\rangle$ of a quantized harmonic oscillator satisfies the state transition algebraic relations $\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$; $\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$.

(i) Identify the operators $\hat{a}$ and $\hat{a}^\dagger$ **(2 marks)**

(ii) By evaluating $[\hat{a}, a^\dagger]|n\rangle$, show that $[\hat{a}, \hat{a}^\dagger] = 1$ **(5 marks)**

(iii) Determine the uncertainty in the measurement of the quadrature component $\hat{x}_1$ in the number state $|n\rangle$, using the definition $\hat{a} = \hat{x}_1 + i\hat{x}_2$; $\hat{a}^\dagger = \hat{x}_1 - i\hat{x}_2$ **(8 marks)**

QUESTION 3 (20 MARKS)

a) Explain the Heisenberg picture of Quantum mechanics, hence derive the Heisenberg's equation of motion. **(8 marks)**

b) Show that the spin-up and spin-down state vectors of an electron satisfy the orthonormality relations $\langle u|u\rangle = \langle d|d\rangle = 1$; $\langle u|d\rangle = \langle d|u\rangle = 0$ **(3 marks)**

c) Angular momentum operators can be defined by the following matrices.

$\hat{L}_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$; $\hat{L}_y = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$; $\hat{L}_z = \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$. Determine the commutation brackets $[\hat{L}_x, \hat{L}_y]$, $[\hat{L}_y, \hat{L}_z]$ and $[\hat{L}_z, \hat{L}_x]$ in terms of $\hat{L}_x$, $\hat{L}_y$ and $\hat{L}_z$

(9 marks)

QUESTION 4 (20 MARKS)

a) Using $u(r) = rR(r)$, the radial equation for a one-electron atom is obtained in the form $\left(-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{\hbar^2 l(l+1)}{2\mu r^2} - \frac{Ze^2}{4\pi\varepsilon_0 r}\right)u(r) = Eu(r)$ where the symbols have their usual meanings.

(i) By completing the square of the effective potential, determine the

quantized orbit energy in the form $E_{l+1} = \frac{-\mu Z^2 e^4}{2(4\pi\varepsilon_0)^2\hbar^2(l+1)^2}$. **(5 marks)**

(ii) Show that the radial equation can be factorized in the form

$\left(\frac{d}{dr} + K_{l+1}(r)\right)\left(-\frac{d}{dr} + K_{l+1}(r)\right)u(r) = \frac{2\mu}{\hbar^2}(E - E_{l+1})u(r)$ where the parameter $K_{l+1}(r)$

must be defined in the derivation. **(5 marks)**

b) A particle moves in the one-dimensional potential defined by $V(x) = \begin{cases} V_0 \cos\left(\frac{\pi x}{2a}\right) & |x| \le a \\ \infty & |x| > a \end{cases}$

By treating the potential as a perturbation, obtain the first order energy correction, given that the unperturbed eigen function is

$$u_n = \begin{cases} \frac{1}{\sqrt{a}}\cos\left(\frac{n\pi x}{2a}\right) & n = odd \\ \frac{1}{\sqrt{a}}\sin\left(\frac{n\pi x}{2a}\right) & n = even \end{cases}.$$

(10 marks)

QUESTION 5 (20 MARKS)

a) A two-level system described by the wave function

$\psi(t) = c_a(t)\psi_a e^{-iE_a\frac{t}{\hbar}} + c_b(t)\psi_b e^{-iE_b\frac{t}{\hbar}}$ experiences a time-dependent perturbation. Suppose the system is in state ψ_a intially, derive the expressions for the first order approximations of probability amplitudes, $c_a^1(t)$ and $c_b^1(t)$. **(12 marks)**

b) If the perturbation in 5 (a) above is of the form $H_{ab}^I = V_{ab}\cos\omega t$, show that the transition probability is given by $P_{a\to b} = \frac{|V_{ab}|^2}{\hbar^2}\frac{\sin^2(\omega_0 - \omega)\frac{t}{2}}{(\omega_0 - \omega)^2}$ **(8 marks)**