# JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY <br> SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE <br> ACTUARIAL <br> 3 RD YEAR $1^{\text {ST }}$ SEMESTER 2019/2020 <br> REGULAR (MAIN) 

COURSE CODE: SAC 302
COURSE TITLE: METHODS OF ACTUARIAL INVESTIGATION II
EXAM VENUE: STREAM: (BSc Actuarial Science)
DATE: 1/12/20
EXAM SESSION: 3-6 PM
TIME: 3.00 HOURS

## Instructions:

1. Answer question 1 (Compulsory) and ANY other 2 questions
2. Candidates are advised to write on the text editor provided, or to write on a foolscap, scan and upload alongside the question.
3. Candidates must ensure that they submit their work by clicking 'FINISH AND SUBMIT ATTEMPT' button at the end.

## QUESTION ONE (30 MARKS)

a) 10,000 school children have been selected to take part in a one year medical study. If the initial annual rate of mortality is 0.00025 for each child and deaths are expected to occur independently, calculate the probability that 2 or more of the participants will die before the end of the study.
b) Using the Balducci assumption, derive an expression for $\mathrm{t}_{\mathrm{x}}$ in terms of $q_{x}$.
c) Show that the Balducci assumption implies a decreasing force of mortality between integer ages.
(3 Marks)
d) A small country involved in a war conscripted a cohort of healthy young men to serve in the country's army for a 3-year period starting on 1 January 1999. During this period a number of the men were killed. Given that the total period of service for the group as a whole was 10 million man-days and that the annual force of mortality for death in active service is 0.02 , use a normal approximation to calculate the probability that at least 500 men were killed while in active service.
(5 Marks)
e) Define the following terms and phrases as used in Actuarial investigations.
i) Rate Interval
ii) Age Last Birthday
iii) Age Nearest Birthday
iv) Age Next Birthday
v) Exposed to Risk
(5 Marks)
f) State The principle of correspondence
(2 Marks)
g) Describe the following statistical tests of crude estimates, for comparison with a standard table:
i.) Chi-Square Test
ii.) Standardised Deviations Test
iii.) Sign Test
iv.) Cumulative Deviation Test
v.) Grouping of Signs Test
vi.) Serial Correlations Test
h) What would be the major problem of charging premiums that are:
i.) too low?
ii.) too high?

## QUESTION TWO (20 MARKS)

a) A student has said "If the data includes the whole population, there is no need to graduate the crude rates because there will be Non Sampling Errors". Discuss briefly.(3 Marks)
b) Consider the test:
$H_{0}$ : Smoking has no effect on mortality, versus
$H_{1}$ : Smoking increases mortality
i.) Is this a One-Sided or Two-Sided test?
ii.) What are the possible conclusions of this test?
c) A study of causes of death in elderly men in the 1970s showed the proportions given in the table below. Carry out a chi square test to determine whether these percentages can still be considered to provide an accurate description of causes of death in 2003.

| Cause of death | Proportion of <br> deaths in 1975 | Number of <br> deaths in 2003 |
| :--- | :---: | :---: |
| Cancer | $8 \%$ | 286 |
| Heart disease | $22 \%$ | 805 |
| Other circulatory disease | $40 \%$ | 1,548 |
| Respiratory diseases | $19 \%$ | 755 |
| Other causes | $11 \%$ | 464 |

## QUESTION THREE (20 MARKS)

a) The mortality rates for a population for the age range 30-34 were estimated by fitting a straight line $\alpha+\beta x$ to the crude values of $\log _{e}\left(q_{x} / p_{x}\right)$. Test whether this model (with estimated parameter values of $\alpha=10.9446$ and $\beta=0.110404$ ) can be considered to give a good fit to the data shown in the table below for 2003.

| Age x | 30 | 31 | 32 | 33 | 34 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Number of <br> deaths in 2003 | 335 | 391 | 428 | 436 | 458 |

The initial exposed to risk in 2003 was approximately 700,000 at each age
(10 Marks)
b) A graduation covers 20 age groups and has resulted in 6 positive and 14 negative deviations. Carry out a signs test on these data.
( 5 Marks)
c) A large computer company always maintains a workforce of exactly 5,000 young workers, immediately replacing any worker who leaves. Calculate the probability that there will be fewer than 3 deaths during any 6 month period, assuming that all workers experience a constant force of mortality of 0.0008 per annum.
(5 Marks)

## QUESTION FOUR (20 MARKS)

a) The population of a small town is expected to remain constant at 50,000 over the next few years. Assuming that all inhabitants experience a force of mortality of 0.001 per annum, use a normal approximation to estimate the probability that there will be more than 225 deaths in the town during the next 4 years.
b) A large computer company always maintains a workforce of exactly 5,000 young workers, immediately replacing any worker who leaves. Calculate the probability that there will be fewer than 3 deaths during any 6 month period, assuming that all workers experience a constant force of mortality of 0.0008 per annum. (10 Marks)

## QUESTION FIVE (20 MARKS)

a) Explain how the following factors may influence mortality rates:
i.) sales channel (consider a mailshot to selected existing policyholders and an advert in a popular tabloid national newspaper)
(2 Marks)
ii.) occupation of policyholder (consider a deep-sea diver, a high-street newspaper vendor and an actuary).
( 2 Marks)
b) A mortality investigation covers the period 1 January 2001 to 31 December 2003. In this investigation, the age label used is "age last birthday". Give the range of dates for which the lives in the following table contribute to $E_{x}^{c}$ at each age where they make a contribution. Assume that the day of entry counts in the exposed to risk but the day of exit does not.

|  | Date of birth | Date of joining | Date of exit | Reason for exit |
| :--- | :--- | :--- | :--- | :--- |
| A | 25.04 .69 | 07.08 .99 | 30.10 .02 | Death |
| B | 01.07 .69 | 12.09 .02 | --- | --- |
| C | 04.09 .68 | 22.07 .03 | 4.12 .03 | Withdrawal |

