



**JARAMOGI OGINGA ODINGA UNIVERSITY OF
SCIENCE AND TECHNOLOGY**

**UNIVERSITY EXAMINATIONS
SPECIAL RESIT 2020/2021 ACADEMIC YEAR**

SCHOOL OF MATHEMATICS, ACTUARIAL SCIENCES BPS

SEMESTER ONE, FIRST YEAR EXAMINATIONS for BSc/BEd

SUPPLEMENTARY/SPECIAL

SMA103: LINEAR ALGEBRA 1

Nov, 2020

Time: 2hrs

INSTRUCTIONS

Answer **Question1** and two other questions

Show all the necessary working

Question1 [30marks] Compulsory

(a) Define the vector subspaces H_1, H_2 of vector space R^3 by , $H_1 = \{(x, y, z) : x + 2y + 2z = 0\}$,
 $H_2 = \{(x, y, z) : 2x + 2y - 8z = 0\}$.

(i) Verify that both H_1, H_2 do contain the zero vector. [6 marks]

(ii) Find bases for H_1, H_2 . [6 marks]

(b) Suppose the mapping $L : R^3 \rightarrow R^3$ with $L \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x - y - 8z \\ x + 2y + z \\ 6z \end{bmatrix}$

(i) Show that L is linear. (ii) Determine $\ker(L)$ and $\text{Im}(L)$. [9 marks]

(c) Given the system of linear equations

$$2x + y = 700$$

$$5x + 3y = 20$$

(i) express it in the matrix form $A\tilde{X} = \tilde{b}$

(ii) apply the elementary matrix row reduction operations on the associated augmented matrix;

$A : I : \tilde{b}$ to reduce to the final form $I : \hat{A} : \hat{\tilde{b}}$ where I is the two by two identity matrix.
Compute matrix products $A\hat{A}$, $\hat{A}A$ and hence obtain A^{-1} and \tilde{X} . [9 marks]

Question2 [20marks]

(a) Given matrix $M = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$

(i) Show that $M^2 = 4I_{4 \times 4}$ and hence find M^{-1} , the inverse of M .

(ii) Show that the following vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$

are linearly independent.

[11 marks]

(b) Suppose $T: [x, y, z] \rightarrow [x, x-2y, 2y]$. Construct matrix A of linear mapping T with respect to the standard ordered basis for basis for R^3 .

[9 marks]

Question3 [20marks]

(a) Without using direct computation, show that $\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ are eigenvectors of

matrix $A = \begin{pmatrix} 1 & -4 & -4 \\ 8 & -11 & -8 \\ -8 & 8 & 5 \end{pmatrix}$. Give the associated eigenvalues $\lambda_1, \lambda_2, \lambda_3$ of this matrix.

Verify that $trace(A) = \lambda_1 + \lambda_2 + \lambda_3$

[12 marks]

(b) Verify that matrix $B = A^t$

[8 marks]

has same eigenvalues $\lambda_1, \lambda_2, \lambda_3$

Question4 [20marks]

Define a linear mapping T from vector space X into vector space Y i.e. $T: X \rightarrow Y$

(a) Explain what is meant by (i)kernel of T (ii)image of T (iii)rank of T (iv) nullity of T [8 marks]

(b) State the relationship between dimension of kernel of T and rank of T [2marks]

(c) For matrix. $M = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ determine adjoint of M and hence state M^{-1} [10marks]

Question5 [20marks]

Let $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{pmatrix}$ be a matrix of linear transformation T .

(a) Determine kernel of T [6marks]

(b) Determine range of T [7marks]

(c) State nullity and rank of T [7marks]