

ABSTRACT

The advection-diffusion equation of the form: $U_t = aU_{xx} + cU_x$ where a and c are constants were widely used for modeling and simulation of various complex phenomena in science and engineering. Since for most application problems it was not easy to solve advection-diffusion equation analytically, efficient numerical algorithm have since become important to numerical simulation involving advection-diffusion equation and their stability analysis. The advection term cU_x describes how the substance is carried along by the fluid for example oil carried on the surface of sea water due to currents produced by the wind and tides. The diffusion term aU_{xx} describes the spreading of a substance due to random molecule collision or due to large forces such as vorticity or turbulence. The equation had been solved for $a = 1$ and $c = 0.5, 1, 2$; $a = 0.5, 1, 1.5$ and $c = 0.5, 1, 2$ respectively; $a = 1$ and $c = 0, 4$; $a = 0$ and $c = 1$; However, it has been shown that a solution does not exist for a large advection co-efficient $c = 5$ and $a = 0.5$. It had also been established that for a flow to take place from left to right, the advection and diffusion co-efficient had to be greater than zero. Analytic techniques so far used were limited in capturing the complex boundary shapes generated by the equation. Attempts were made to numerically solve the advection-diffusion equation with co-efficient put at between $0 \leq a \leq 1$ and $0 \leq c \leq 1$. Other attempt had put $0 \leq a \leq 1$ and $c = 2, 4, \text{ and } 5$ but curiously left out the case when $c = 3$. Since all the above mentioned attempts varied the advection co-efficients between $0 \leq c \leq 2$ and $c = 4$ and 5 , there was need to investigate the nature of the solution when $a = 1$ and $c = 3$. The objective of this study was therefore to numerically solve this equation using finite difference method when the advection co-efficient $c = 3$ and the diffusion co-efficient $a = 1$ and to analyse the stability of the resulting numerical schemes. We have discretized the advection-diffusion equation using the finite difference methods i.e Crank-Nicolson and Pure Implicit Schemes which were stable on the condition of r and h (mesh ratio and distance). The results obtained from this study (using Matlab) can be interpreted without difficulty and may be used in the analysis and disposal of nuclear waste, transportation of nutrients in trees, study of oceanography, study of marine biology and the study of earthquake.