



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR BACHELOR OF ACTUARIAL SCIENCE

APRIL 2022/2023

MAIN REGULAR

COURSE CODE: WAB 2103

COURSE TITLE: Mathematical Modelling

EXAM VENUE: LAB 17

STREAM: B.Sc. Actuarial Science

DATE:14/12/2022

EXAM SESSION: 15.00-17.00PM

TIME: 2 HOURS

Instructions:

- 1. Answer ONE and any other two questions only.**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE COMPULSORY (30 MARKS)

- a) Explain the following giving an example in each case (8mks)
- i) Difference equations
 - ii) Differential equations
 - iii) Homogeneous difference equations
 - iv) The order of a difference equation
- b) It is known that, for certain income brackets, money deposited in an individual retirement Account (IRA) and its interest are tax-deferred. Karen Smith sets up an IRA at the beginning of the year and deposited \$3,000. Karen will deposit \$2,000 at the end of each year, and the interest rate is 6% compounded annually.
- i) Model this situation by a difference equation. (3mks)
 - ii) What will be the future value of the account at Karen's retirement age of 60 if she starts the IRA at the age of 25? (5mks)
- c) Solve the differential equation $y' = e^{x-y}$. (3mks)
- d) Find the interest on a 60- day loan of \$1500 at 4.15% (5mks)
- e) Solve the following difference equations to find x_{10}
- i) $x_{n+1} = 1.8x_n + 10, x_0 = 20$ (3mks)
 - ii) $x_{n+1} - x_n = 3x_n + 4, x_0 = 2$ (3mks)

Question Two(20mks)

- a) Assume you open an account that pays 4% compounded annually and deposit \$2,000. You will deposit 2% more into your account than you deposited in the previous year.
- i) Model this situation by a difference equation. (4mks)
 - ii) What is the total amount in the account after 20 years? (6mks)
- b) You just won the lottery. You put your \$5,000,000 in winnings into a fund that has a rate of return of 4%. Each year you use \$300,000. Calculate the amount of money will you will have twenty years from now? (10mks)

Question Three(20mks)

- a) A population of insects in a region will grow at a rate that is proportional to their current population. In the absence of any outside factors the population will triple in two weeks time. On any given day there is a net migration into the area of 15 insects and 16 are eaten by the local bird population and 7 die of natural causes. If there are initially 100 insects in the area will the population survive? If not, when do they die out? (10mks)

- b) Find the solution to the following differential equation.

$$\frac{dv}{dt} = 9.8 - 0.196v \quad (10mks)$$

Question Four(20mks)

- a) Your parents are considering a 30 year, \$100000 mortgage that charges 0.5% interest each month. They want to pay \$p monthly so that the mortgage (loan) is paid off after 360 payments (i.e., 30 years).

- i) Set up and solve the model with the dynamical system (i.e., difference equation).
ii) Find the monthly payment \$p. (10mks)

- b) A 1500 gallon tank initially contains 600 gallons of water with 5 lbs of salt dissolved in it. Water enters the tank at a rate of 9 gal/hr and the water entering the tank has a salt concentration of $15(1+\cos(t))$ lbs/gal. If a well-mixed solution leaves the tank at a rate of 6 gal/hr, calculate the amount of salt in the tank when it overflows (10mks)

Question Five(20mks)

A runner in the Ndakaine Marathon starts out too fast; as consequence her speed decreases throughout the race at a rate inversely proportional to the square root of time.

- (a) Set up the differential equation that describes how her speed depends on time. (5mks)
(b) The runner starts out running 8 mph, and after one hour her speed has reduced to 7 mph. Find her speed as a function of time. (10mks)
(c) Find the function that expresses her distance from the starting line as a function of time. (5mks)

