

$$\text{div}\vec{D} = \vec{\rho}_f$$

$$\text{div}\vec{B} = 0$$

$$\text{curl}\vec{E} = -\frac{\delta\vec{B}}{\delta t}$$

$$\text{curl}\vec{H} = \vec{J}_f + \frac{\delta\vec{D}}{\delta t}$$

Explain briefly the significance of the equations (4 mks)

(d) (i) The radially dependent volume charge density $\rho_v = 50r^2 \text{ C/m}^3$ exists within a sphere of radius $r = 0.05 \text{ m}$. Find the total charge q contained by the sphere (2 mks)

(ii) The same sphere of (i) is now covered with the angularly dependent surface charge density $\sigma = 2 \times 10^{-2} \cos^2 \theta \text{ C/m}^2$. Find the total charge q contained by that sphere (2 mks)

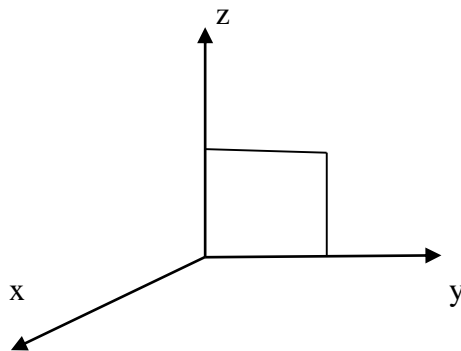
(e) (i) What is meant by Gauge transformations ? (2 mks)

(ii) Give a mathematical expression of Lorentz condition and its advantage (3 mks)

(f) (i) Explain what is meant by a transmission line. Describe briefly the classification of transmission lines (3mks)

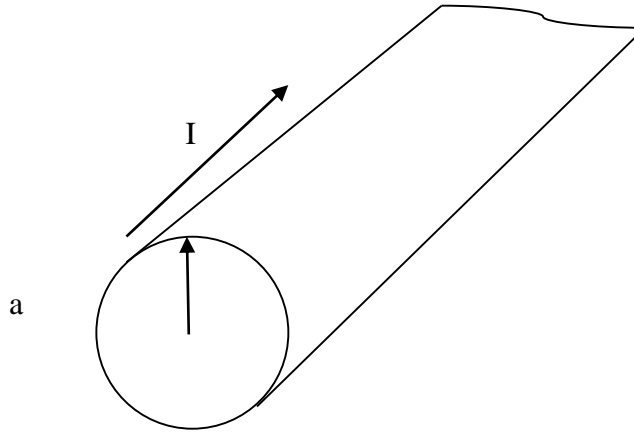
(ii) Draw a flow chart showing the steps to develop a method for transmission of transverse electric waves in a hollow rectangular waveguide (3mks)

(g) Suppose $v = (2xz + 3y^2)\vec{j} + (4yz^2)\vec{k}$. Check Stokes' theorem for the square surface shown in the figure below (5 mks)



QUESTION TWO

(a) A cylindrical conductor of radius a and conductivity σ carries a steady current I which is distributed uniformly over its cross-section, as shown in the figure below



- (i) Compute the electric field E inside the conductor (1 mk)
- (ii) Compute the magnetic field B just outside the conductor (3 mks)
- (iii) Compute the Poynting vector S at the surface of the conductor. In which direction does S point? (3 mks)
- (iv) By integrating S over the surface area of the conductor, show that the rate at which electromagnetic energy enters the surface of the conductor is equal to the rate at which energy is dissipated. (5mks)

(b) Suppose the electric field of an electromagnetic wave is given by the superposition of two waves $E = E_o \cos(kz - \omega t) + E_o \cos(kz + \omega t)$. You may find the following identities useful

$$\cos(kz \pm \omega t) = \cos(kz)\cos(\omega t) \pm \sin(kz)\sin(\omega t).$$

- (i) What is the associated magnetic field $B(x,y,z,t)$? (3mks)
- (ii) Use the identity $\cos(a)\sin(a) = (1/2)\sin(2a)$ to show that the energy per unit area per unit time (the pointing vector) transported by this wave is $S = (E_o^2 / c\mu_0)\sin(2kz)\sin(2\omega t)$ (5mks)

QUESTION THREE

(a) (i) Explain what is meant by polarization (1 mk)

(ii) State the causes by which the dielectric polarization may occur in the materials (3mks)

(b) (i) Show that a volume charge density ρ_b and a surface charge density σ_b arising from bound charges of the dielectric are given by $\rho_b = -\nabla \cdot P$ and $\sigma_b = P \cdot n$ (8mks)

(ii) Show that the total bound charge of a polarized dielectric of finite extent is always zero (3mks)

(c) On the basis of functional relationship between the polarization P and the electric field E , classify dielectrics (6mks)

QUESTION FOUR

(a) (i) Using Ampere's law in intergral form, derive the Maxwell's equation

$$\text{curl}H = \vec{J}_f + \frac{\delta \vec{D}}{\delta t} \quad (4\text{mks})$$

(ii) Suppose a magnetic field B is applied to a cube of magnetic material, b m on a side such that magnetization density M is z-directed and varies linearly with x-axis according to $M = a_z 10x$ A/m. Find the magnetization current density J_m in the material, as well as the surface magnetization current. Sketch the bound current fields in the cube (6mks)

(b) (i) Derive the Biot-Savart law to find the magnetic field B of a current carrying conductor. (4 mks)

(ii) Apply Biot – Savart law to determine the magnetic field B of the thin wire of length 2L and carrying a steady current (4mks)

(iii) Prove that $\vec{\nabla} \cdot \vec{B} = 0$ (2mks)

QUESTION FIVE

Maxwell's equations are four mathematical equations that relate the Electric Field (E) and magnetic field (B) to the charge density (ρ) and current density (J) that specify the fields and give rise to electromagnetic radiation.

- i. Derive the *four* Maxwell's equations with sources in free space giving an account of the significance of each equation (12 marks)
- ii. Obtain the Maxwell's equations with sources in macroscopic media (4 marks)
- iii. Obtain the Maxwell's equations in vacuum (4 marks)

