JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION

1ST YEAR 2ND SEMESTER 2024/2025 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: WMB 9312

COURSE TITLE: GROUP THEORY

EXAM VENUE:AH **STREAM: BED**

DATE: 14/4/25 **EXAM SESSION: 15-17.00 HRS**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

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QUESTION ONE (30 marks)

a) i) Determine whether the operation $x * y = x + y + 2$ on $\mathbb{Z}$ is a binary operation (5marks)

ii) Prove that if a binary operation $*$ on a set S has an identity, then the identity is unique (5marks)

b) Verify that $\mathbb{Z}_6 = \{0,1,2,3,4,5\}$ under addition modulo 6 forms a group (5 marks)

c) Determine if $(G = \{1, -1, i, -i\}, \times)$ is a cyclic group (5marks)

d) i) Given permutations $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 2 & 5 & 1 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 4 & 5 & 2 \end{pmatrix}$. Find α^{-1} and $\alpha\beta$ (6marks)

ii) Write the permutations (1243) in S_4 as a product of transpositions and determine its parity (4marks)

QUESTION TWO (20 marks)

a) Define a normal subgroup and provide an example of a normal subgroup in the group of $\mathbb{Z}_6$ (4marks)

b) Let $G = \mathbb{Z}_9$ integers modulo 9 under addition and let Let $H = \{0,3,6\}$.

i) List all the left cosets of H in G (2marks)

ii) List all the right cosets of H in G (2marks)

iii) Are the left cosets the same as the right cosets? Why? (2marks)

c) Let G be a set with the operation of matrix multiplication defined by $G = \left\{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\right\}$. Verify whether G is a group under matrix multiplication (10marks)

QUESTION THREE (20 marks)

a) Let $G = \mathbb{Z}_3$, the cyclic group of order 3 under addition modulo 3, and $X = \{1,2,3\}$. Define the action of G on X by cyclic permutations. Write the orbit of each element in X (10marks)

b) Let $G = \mathbb{Z}_8$(integers modulo 8 under addition) and $H = \{0,4\}$. Verify that H is a normal subgroup of G and construct the quotient group G/H (10marks)

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QUESTION FOUR (20 marks)

a) Let G be a finite group with order $|G| = 12$, and let H be a subgroup of G with order

$|H| = 4$

i) State Lagrenge's Theorem and use it to determine the number of cosets of H in G (5marks)

ii) Show that the order of any subgroup of G divides the order of G (5marks)

b) Determine whether the map $\phi: \mathbb{Z} \to \mathbb{Z}_4$, defined by $\phi(a) = a \bmod 4$, is homormophism. Find its kernel and image
(10marks)

QUESTION FIVE (20 marks)

a) State and prove the first isomorphism theorem for groups (10marks)

b) Explain how group theory can be applied in music, specifically in analysing musical compositions or scales. Provide an example of a musical scale and describe how it relates with a group structure (10marks)

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