



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE
UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE
ACTUARIAL
SPECIAL RESIT 2015/2016 ACADEMIC YEAR
MAIN REGULAR

COURSE CODE: SMA 405

COURSE TITLE: PARTIAL DIFFERENTIAL EQUATION I

EXAM VENUE: LAB 1

STREAM: (BSc. Actuarial/ Education)

DATE: 04/05/2016

EXAM SESSION: 2.00 – 4.00 PM

TIME: 2.00 HOURS

Instructions:

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (COMPULSORY)

a) State the order and degree of the partial differential equations below

i)
$$\frac{\partial^2 y}{\partial x^2} + \left(\frac{\partial y}{\partial x}\right)^3 + \left(\frac{\partial^3 z}{\partial x^3}\right)^4 = 0$$

ii)
$$\left(\frac{\partial y}{\partial x}\right)^4 + \frac{\partial^2 z}{\partial x^2} + \frac{\partial^3 z}{\partial x^3} = 0 \quad (4 \text{ marks})$$

b) Define the following

i) Total differential equation

ii) Non-Linear partial differential Equation

iii) Semi-linear partial differential Equation

iv) Quasi-linear partial differential equation (8 marks)

c) Solve the simultaneous Differential equation

$$\frac{dx}{xz(z^2 + xy)} = \frac{dy}{-yz(z^2 + xy)} = \frac{dz}{x^4} \quad (6 \text{ marks})$$

d) Find the orthogonal trajectory on the cone $x^2 + y^2 = z^2 \tan^2 \alpha$ of its intersection with the family of planes parallel to $z = 0$ (8 marks)

e) Solve the following differential equations by inspection

i)
$$df(x, y) = \frac{xdy + ydx}{x^2}$$

ii)
$$df(x, y) = \frac{xdy + ydx}{x^2 + y^2} \quad (4 \text{ marks})$$

QUESTION TWO

By eliminating the arbitrary constants a and b from $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ form a partial differential equation (4 marks)

a) Solve the homogeneous equation

$$(x^2 y - y^3 - y^2 z)dx + (xy^2 - x^2 z - x^3)dy + (xy^2 + x^2 y)dz = 0 \quad (10 \text{ marks})$$

b) By choosing appropriate multipliers solve

$$\frac{dx}{4y - 3z} = \frac{dy}{4x - 2z} = \frac{dz}{2y - 3x} \quad (6 \text{ marks})$$

QUESTION THREE

a) Solve the Pfaffian differential equation

$$(yz + xyz)dx + (zx + xyz)dy + (xy + xyz)dz = 0 \quad (5 \text{ marks})$$

b) Find $f(y)$ such that the Pfaffian differential equation

$$\{(yz + z)/x\}dx - zdy + f(y)dz = 0 \text{ is integrable hence solve it.} \quad (10 \text{ marks})$$

c) Use Lagrange's method to solve $xyp + y^2 q = zxy - 2x^2$ (5 marks)

QUESTION FOUR

- a) Show that the equation $xp - yq = x$ and $x^2p + q = xy$ are compatible hence find their solution. (10 marks)
- b) Solve $(x^2 + y^2)p + 2xyq = z(x + y)$ (5 marks)
- c) Form a partial differential equation by eliminating the arbitrary function f from the function $x + y + z = f(x^2 + y^2 + z^2)$ (5 marks)

QUESTION FIVE

- a) Solve the Cauchy's problem for $zp + q = 1$ where the initial data curve is $x_0 = \mu, y_0 = \mu, z_0 = \frac{\mu}{2}$ for $0 \leq \mu \leq 1$ (8 marks)
- b) Use Charpit's method to find the complete integral of $p^2 - y^2q = y^2 - x^2$ (12 marks)