



**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**  
**SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE**  
**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE**  
**ACTUARIAL**  
**SPECIAL RESIT 2015/2016 ACADEMIC YEAR**  
**MAIN REGULAR**

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**COURSE CODE: SMA 301**

**COURSE TITLE: ORDINARY DIFFERENTIAL EQUATION I**

**EXAM VENUE:** **STREAM: (BSc. Actuarial)**

**DATE:** **EXAM SESSION:**

**TIME: 2.00 HOURS**

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**Instructions:**

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

### QUESTION ONE (COMPULSORY)

a) Given  $y = A \sin x + B \cos x$ , where  $A$  and  $B$  are arbitrary constants, eliminate the arbitrary constants to form a differential equation hence state its order and degree. (5 marks)

b) The rate of cooling of a body is proportional to the excess of its temperature above its surrounding  $\theta^\circ\text{C}$ . A body cools from  $85^\circ\text{C}$  to  $65^\circ\text{C}$  in 4.0 minutes at a surrounding temperature of  $15^\circ\text{C}$ . Determine how long to the nearest second the body will take to cool to  $55^\circ\text{C}$ . (4 marks)

c) Solve the differential equation below using an appropriate method

$$\frac{d^2 y}{dx^2} + 36y = 0 \quad (5 \text{ marks})$$

d) Using an appropriate method solve the differential equation  $2yy'' = 1 + y'$ . (5 marks)

e) Use the method of variation of parameters to solve  $\frac{d^2 y}{dx^2} - 7\frac{dy}{dx} + 12y = e^{2x}$ . (5 marks)

f) Solve the differential equation  $(y - 2x - 4)dy = (y + 2x - 2)dx$  (6 marks)

### QUESTION TWO (20 marks)

a) By finding the integrating factor, find the general solution of the differential equation

$$\frac{(1-x^2)}{x} \frac{dy}{dx} + \frac{2x^2-1}{x^2} y = x \quad (\text{Hint: Use partial fractions}) \quad (10 \text{ marks})$$

b) A resistance ( $R$ ) of 100 ohms, an inductance ( $L$ ) of 0.5 henry are connected in series with a battery of 20 volts( $V$ ). Find the current ( $i$ ) in the circuit as a function of time( $t$ ) given that they are connected by the differential equation  $Ri + L\frac{di}{dt} = V$ . (5 marks)

c) Solve the differential equation below using any appropriate method

$$4\frac{d^2 y}{dx^2} + 4\frac{dy}{dx} + y = e^x. \quad (5 \text{ marks})$$

### QUESTION THREE

a) Consider a second order differential equation

$$a\frac{d^2 y}{dx^2} + b\frac{dy}{dx} + cy = F(x)$$

Let  $F(x) = 0$  and let  $y = U$  and  $y = V$ , where  $U$  and  $V$  are functions of  $x$  be two solutions to the differential equation, then show that  $y = (U + V)$  is also a solution. (6 marks)

b) Find the general solution of the differential equations

$$(i) (xy - x^2)dy + y^2 dx = 0 \quad (4 \text{ marks})$$

$$(ii) \frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + 5y = 0 \quad (4 \text{ marks})$$

$$(iii) \frac{d^2 y}{dx^2} - 36y = 2\cos 4x \quad (4 \text{ marks})$$

#### QUESTION FOUR

Use any appropriate method to solve each of the differential equations below

- a)  $(2 - 9xy^2)xdx + (4y^2 - 6x^3)ydy = 0$  given that  $y = 4$  when  $x = 1$ . (6 marks)
- b)  $yy'' + (y')^2 = 0$  (6 marks)
- c)  $\frac{dy}{dx} + \frac{x}{1-x^2}y = x\sqrt{y}$  (8 marks)

#### QUESTION FIVE

- a) Detectives discovered a murder victim at 6.30 am and the body temperature of the victim was then  $26^\circ\text{C}$ . After 30 minutes the police surgeon arrived and found the body temperature to be  $23^\circ\text{C}$ . If the air temperature was  $16^\circ\text{C}$  throughout and the normal body temperature is  $37^\circ\text{C}$ . At what time did the police surgeon estimate that the crime occurred. (10 marks)
- b) Solve the differential equation  $xy'' = y' + (y')^3$  given  $x = 1$  when  $y = 0$  and  $x = 2$  when  $\frac{dy}{dx} = 1$  (10 marks)