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SPECIAL RESIT EXAMINATION-APRIL/MAY 2016

SMA 208: INTRODUCTION TO ANALYSIS

INSTRUCTION: Attempt question one (**COMPULSORY**) and any other TWO questions only.

QUESTION ONE(*COMPULSORY*) [30 MARKS]

(a) Define the terms: Ordered field, Supremum and Infimum. (6 marks)

(b) Given that $S = \{x : 0 < x \leq 1, x \in \mathbb{R}\}$, determine the infimum and the supremum and state whether they belong to S or not. (3 marks)

(c) Let $a, b \in \mathbb{R}^+$. Show that there exists a positive integer n such that $na > b$. (5 marks)

(d) State **without proof**, the Dedekind's form of completeness property. (3 marks)

(e) Show that if M and N are neighbourhoods of a point x then $M \cap N$ is also a neighbourhood of x. (6 marks)

(f) Show that every open set is the union of open intervals. (4 marks)

(g) Given a sequence $\{a_n\}$, show that $\overline{\lim} a_n = +\infty$ if and only if $\{a_n\}$ is not bounded above. (3 marks)

QUESTION TWO [20 marks]

(a) If A and B are two open sets, show that their union is also open. (5 marks)

(b) Given a nonempty set S, show that its' supremum is unique. (7 marks)

(c) Show that there is no rational number whose square is 5. (8 marks)

QUESTION THREE [20 marks]

(a) Define the absolute value function. (2 marks)

(b) Given that $x, y \in \mathbb{R}$, show that:

(i) $|x|^2 = x^2 = |-x|^2$. (3 marks)

(ii) $|x + y| \leq |x| + |y|$. (3 marks)

(c) Distinguish between Range of a function and Composite function. (4 marks)

(d) Given that $f(x) = \frac{x^3-1}{x-1}$, find its limit as $x \to 1$. (5 marks)

(e) Show that $f(x) = -x$ is one to one and onto for all $x \in \mathbb{N}$. (3 marks)

QUESTION FOUR [20 marks]

(a) Explain the following concepts: (i) Limit Inferior and limit superior; (ii) Convergence of a sequence. (6 marks)

(b) Show that

$$\lim_{n\to\infty} \frac{3 + 2\sqrt{n}}{\sqrt{n}} = 2.$$

(6 marks)

(c) Show that every bounded sequence has a limit point. (8 marks)

QUESTION FIVE [20 marks]

(a) If P and Q are open sets, show that $(P \cap Q)^c = P^c \cup Q^c$. (4 marks)

(b) Show that a set is closed if its' complement is open. (7 marks)

(c) Show that the set of rational numbers is not order complete. (9 marks)