



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL
SCIENCES
UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION
SPECIAL RESITS DECEMBER 2022
MAIN REGULAR

COURSE CODE: WMB9312

COURSE TITLE: GROUP THEORY

EXAM VENUE:

STREAM: (EDUCATION)

DATE: 30/11/2022

EXAM SESSION: 9.00-11.00AM

TIME: 2.00 HOURS

Instructions:

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 marks)

- a) Define the following terms
- i) A Homomorphism. (2marks)
 - ii) A Quotient Subgroup. (2marks)
- b) Let $(G,*)$ be a group. If $x, y, z \in G$ with $x * z = y * z$, then show that $x = y$. (4marks)
- c) Let $(G,*)$, (H, \dagger) be groups and let $f : G \rightarrow H$ be a homomorphism. Then show that $f(e_G) = e_H$, where e_G and e_H are identity elements in $(G,*)$ and (H, \dagger) respectively. (4marks)
- d) Determine the order of
- i) 2 in $(\mathbb{Z}_5, +)$.
 - ii) $(\mathbb{Z}_4, +)$.
 - iii) $(\mathbb{R} \setminus \{0\}, \times)$. (4marks)
- e) Determine the generators of $\mathbb{Z}_5 \setminus \{0\}$ under multiplication. (5marks)
- f) Is the set $\{-2, -1, 0, 1, 2, \dots\}$ a subgroup of $(\mathbb{Z}, +)$? (4marks)
- g) Find the signature of the permutation $\sigma = (1, 6)(2, 5)(3, 4)$. (5marks)

QUESTION TWO (20 marks)

- a) Below is a partially filled Cayley table.

*	a	b	c
a			
b			b
c			

- i. Fill in the missing parts. (4marks)
 - ii. Determine the Identity Element of the Group. (1mark)
 - iii. Determine the Inverse of each element of the group. (3marks)
- b) Determine whether $(\mathbb{Z}_4, +)$ is a group or not. (10marks)
- c) State the First Isomorphism Theory. (2marks)

QUESTION THREE (20 marks)

- a) Given that $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 4 & 6 & 5 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 6 & 5 & 2 \end{pmatrix}$.
- i) Show that $\sigma \circ \tau \neq \tau \circ \sigma$. (6marks)
 - ii) Find σ^{-1} . (2marks)
 - iii) Write both σ and τ in cycle notation. (4marks)
 - iv) Draw the graph of σ . (6marks)
- b) Define a group action. (2marks)

QUESTION FOUR (20 marks)

- a) Let $(G, *)$, $(H, †)$ and $(K, ‡)$ be groups and let $f : G \rightarrow H$ and $g : H \rightarrow K$ be homomorphisms. Then show that
- i) $g \circ f : G \rightarrow K$ is also a homomorphism. (4marks)
 - ii) If $f : G \rightarrow H$ is an isomorphism, then $f^{-1} : H \rightarrow G$ is also an isomorphism. (6marks)
 - iii) If f and g are isomorphisms, so is $g \circ f$. (6marks)
- b) Determine the subgroups of $(\mathbb{Z}_4, +)$. (4marks)

QUESTION FIVE (20 marks)

- a) Let $(G, *)$, $(H, †)$ be groups and let $f : G \rightarrow H$ be a homomorphism.
- i) Define the image and the Kernel of f . (4marks)
 - ii) Prove that the image of f is a subgroup of $(H, †)$. (6marks)
- b) Let $(G, *)$ be a group and H be a subgroup. Then show that H is normal if and only if, for each $x \in G$, then the left coset is equal to the right coset. (6marks)
- c) Prove that the relation of isomorphism between groups is an equivalence relation. (4marks)