



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES
UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION
SCIENCE WITH IT
3RD YEAR 1ST SEMESTER 2016/2017 ACADEMIC YEAR
MAIN CAMPUS

COURSE CODE: SPH 302

COURSE TITLE: THERMODYNAMICS

EXAM VENUE: LAB 9

STREAM: (BED Sc.)

DATE: 20/04/16

EXAM SESSION: 9.00 – 11.00 AM

TIME: 2 HOURS

Instructions:

- 1. Answer question 1 (compulsory) and ANY other 2 questions.**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Universal gas constant (R)	8.314 J/Mol.K
Boltzmann constant (k)	1.381×10^{-23} J.K ⁻¹ .
Specific heat of water (c)	4200 J/Kg.K
Specific latent heat of fusion of ice (L)	3.34×10^5 J/kg
Adiabatic index of adiatomic gas (γ)	1.4

QUESTION ONE (COMPULSORY)

(a) Starting from the **first law of thermodynamics**, show that the heat absorbed ΔQ for an isothermal expansion of one mole an ideal gas is given by

$$\Delta Q = RT \ln \frac{V_2}{V_1} \quad (3 \text{ marks})$$

(b) Calculate the heat capacity of an **ideal monatomic gas per mole**

(i) at a constant volume C_V (3 marks)

(ii) at a constant pressure C_p (3 marks)

(c) Using the Maxwell's relations, find expression for $(\partial C_V / \partial V)_T$ in terms of p, V and T. (5 marks)

(d) In an adiabatic expansion of an ideal gas, show that

(i) $TV^{\gamma-1} = \text{constant}$ (3 marks)

(ii) $pV^\gamma = \text{constant}$. (2 marks)

(e) What is the adiabatic index, γ for an **ideal monatomic gas**? (3 marks)

(f) Show that the work done when a gas expands adiabatically from V_b to V_c in a Carnot engine

is given by
$$W_{ad} = \frac{P_c P_c - P_b P_b}{1 - \gamma}$$
 Where W_{ad} is adiabatic work

(4 marks)

(g) Derive the **Maxwell relation** based on **Internal energy U**. (4 marks)

QUESTION TWO (20 marks)

(a) State the **first law of thermodynamics** (1 mark)

(b) The internal energy of a non ideal gas is a function of temperature and volume. Show that the heat capacity

(i) at a constant volume C_V is given by $C_V = \left(\frac{\partial U}{\partial T}\right)_V$

(ii) at a constant pressure C_p is given by $C_p = \left(\frac{\partial U}{\partial T}\right)_V + \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right] \left(\frac{\partial V}{\partial T}\right)_P$ (10 marks)

(c) Show that the difference in **molar heat capacity per pressure** and **molar heat capacity per volume** of an ideal monatomic gas is given by $C_p - C_V = R$ (6 marks)

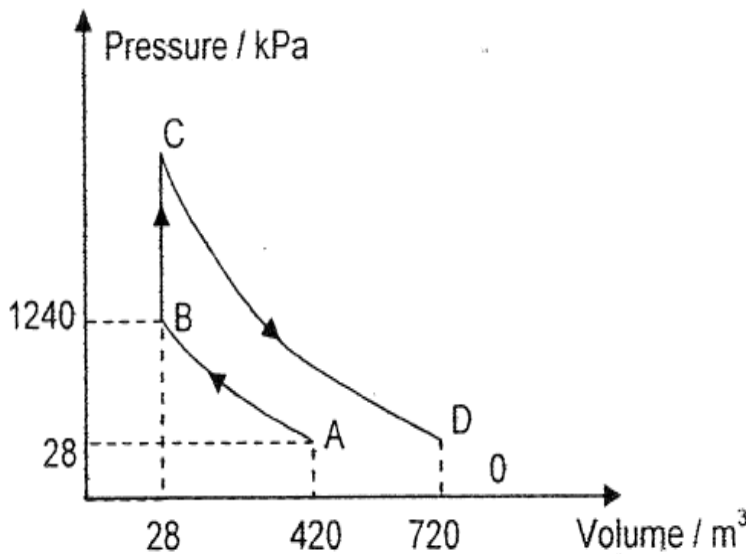
(d) Is it always true that $dU = C_V dT$? (3 marks)

QUESTION THREE (20 marks)

(a) Using a T-S diagram find an expression for Q_h/Q_l for an ideal gas undergoing a Carnot cycle in terms of the temperatures T_h and T_l (3 marks)

(b) The thermodynamics of an engine is illustrated by the graph below. Air enters the engine at a rate of $420 \text{ m}^3/\text{s}$ when the aircraft is cruising at 250 m/s at an altitude of 9000 m .

The air at this altitude has a pressure of 28000 Pa and a temperature of 225 K . The air first passes through a compressor. This reduces the volume of gas to one-fifteenth of its original volume. The compressed air then has fuel continuously sprayed into it. The fuel burns creating a very high pressure and a high temperature gas that expands rapidly out of the rear of the engine. In doing so it does work on the turbine.



Indicator diagram for a Turbofan engine

- i. A→B is an adiabatic compression. If the gas is adiabatic, find the pressure after compression. (2 marks)
- ii. Determine the temperature at B. (2 marks)
- iii. Fuel is burnt to raise the temperature of the air at C to 1400 K . Determine the pressure at C. Assume that there is a constant amount of air throughout the cycle; that is, ignore the chemical processes that can alter the number of moles of gas present. (2 marks)
- iv. The exhaust gases then expand adiabatically to atmospheric pressure. Determine their temperature at D if they have then expanded to 720 m^3 . (2 marks)
- v. Deduce the efficiency of the engine in practice (2 marks)

(c) A large reservoir at temperature T_R is placed in thermal contact with a small system at temperature T_S . They both end up at the temperature of the reservoir, T_R . The heat transferred from the reservoir to the system is $\Delta Q = C(T_R - T_S)$, where C is the heat capacity of the system. Calculate the total entropy change in the Universe. (4 marks)

(d) Consider two systems, with pressures p_1 and p_2 and temperatures T_1 and T_2 . If internal energy ΔU is transferred from system 1 to system 2, and volume ΔV is transferred from system 1 to system 2, find the change of entropy. Show that equilibrium results when $T_1 = T_2$ and $p_1 = p_2$. (3 marks)

QUESTION FOUR (20 marks)

(a) Using the first law $dU = TdS - PdV$ to provide a reminder, write down the definitions of the four **thermodynamic potentials** U, H, F, G (in terms of U, S, T, p, V), and give dU, dH, dF, dG in terms of T, S, p, V and their derivatives. (8 marks)

(b) Show that **Gibbs–Helmholtz equations** are given by

$$U = -T^2 \left(\frac{\partial}{\partial T} \right)_V \frac{F}{T} \text{ and}$$

$$H = -T^2 \left(\frac{\partial}{\partial T} \right)_P \frac{G}{T} \quad (3 \text{ marks})$$

(c) Derive the **Maxwell relation** based on **Gibbs function, G**. (4 marks)

(d) Show that the entropy of an ideal gas increases with increasing temperature and increasing volume. (5 marks)

QUESTION FIVE (20 marks)

(a) Explain three **consequences** of the third law of thermodynamics (6 marks)

(b) Derive the **Clausius–Clapeyron equation** $\frac{dP}{dT} = \frac{L}{T(V_2 - V_1)}$ (5 marks)

(c) Derive an equation for the phase boundary of the liquid and gas phases under the assumptions that the latent heat L is temperature independent, that the vapour can be treated as an ideal gas, and that $V_{\text{vapour}} \gg V_{\text{liquid}}$. (4 marks)

(d) Evaluate the temperature dependence of the latent heat along the phase boundary in a liquid–gas transition and hence deduce the equation of the phase boundary including this temperature dependence. (5 marks)