



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

**UNIVERSITY EXAMINATION FOR THE DEGREE IN BSC. ACTUAL SCIENCE,
BED SCI./ ARTS
SPECIAL RESIT 2020/2021 ACADEMIC YEAR**

COURSE CODE: SMA 300
COURSE TITLE: REAL ANALYSIS
EXAM VENUE:
STREAM: BSC. ACTUARIAL, BED SCI/ARTS

DATE:..... **EXAM SESSION**

TIME: 2 HOURS

Instructions:

- 1. Answer all questions in Section A and any other 2 questions in Section B**
- 2. Candidates are advised not to write on the question paper**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room**

SECTION A

QUESTION ONE compulsory (30 MARKS)

- a) Define the following terms: (6 marks)
- (i) Metric space
 - (ii) Riemann integrable function
 - (iii) Sequence
- b) Find a simple expression for n^{th} terms of each sequence (4 marks)
- i) $1, \frac{1}{2}, \frac{1}{3}, \dots$
 - ii) $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \dots$
- c) Show that $\lim_{n \rightarrow \infty} \left\{ \frac{2n+1}{n+1} \right\} = 2$ (5 marks)
- d) Let (x_n) and (y_n) be two converging sequences as $n \rightarrow \infty$ with the limits l and h respectively. Show that $x_n + y_n \rightarrow l + h$. (7 marks)
- e) Show that $d(\mathfrak{R}, d)$ is a metric space whenever $d(x, y) = |x - y|, x, y \in \mathfrak{R}$ (8 marks)

SECTION B

QUESTION TWO (20 MARKS)

- a) Suppose $f : [a, b] \rightarrow \mathfrak{R}$ is bounded. Prove that $\int_{-a}^b f \leq \int_a^{-b} f$. (6 marks)
- b) If f is a Riemann integrable function on $[a, b]$. Show that $U(f, P) - L(f, P) < \varepsilon$ (10 marks)
- c) Using Bolzano-Weierstrass theorem show that the bounded sequence below is convergent. (4 marks)
- $$\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \dots$$

QUESTION THREE (20 MARKS)

- a) List the first three terms of the sequence (2 marks)
- (i) $(-1)^n n$
 - (ii) $\frac{n+1}{n^2}$
- b) Prove that sequence a_n has at-most one limit. (8 marks)
- c) State and prove monotone convergence theorem (10 marks)

QUESTION FOUR (20 MARKS)

- a) Distinguish between limit inferior and limit superior. (4 marks)
- b) Show that every Cauchy sequence is bounded. (8 marks)
- c) Prove that the sequence $\left(\frac{1}{2^n}\right)$ is Cauchy sequence. (8 marks)

QUESTION FIVE (20 MARKS)

- a) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ converges to 1 (4 marks)
- b) State the following theorems and Lemma (8 marks)
- i) Comparison test theorem
 - ii) Integral test theorem
 - iii) Bolzano-Weirstrass theorem
 - iv) Squeeze Lemma
- c) Prove that the set of real numbers in the interval (0,1) is uncountable. (8 marks)