



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE

ACTUARIAL

4TH YEAR SPECIAL RESITS – 2016

MAIN REGULAR

COURSE CODE: SMA 102

COURSE TITLE: CALCULUS I

EXAM VENUE:

STREAM: (BSc. Actuarial)

DATE:

EXAM SESSION:

TIME: 2.00 HOURS

Instructions:

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (COMPULSORY) (30 marks)

a) Define $\lim_{x \rightarrow a} f(x) = L$. (2 marks)

b) Find the limit (if it exists)

$$\lim_{x \rightarrow -4} \frac{\sqrt{x^2 + 9} - 5}{x + 4} \quad (4 \text{ marks})$$

c) Find the one-sided limits $\lim_{x \rightarrow 2^+} f(x)$ and $\lim_{x \rightarrow 2^-} f(x)$ if

$$f(x) = \begin{cases} 7x - 2 & \text{if } x \geq 2 \\ 3x + 5 & \text{if } x < 2 \end{cases} \quad (4 \text{ marks})$$

d) Determine the point of discontinuity (if any) of the function $f(x)$

$$f(x) = \frac{4x^2 - 11x - 3}{x - 3}$$

If the discontinuity is removable, define the function to make it continuous. (4 marks)

e) Given that $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$, find $f'(x)$ if $f(x) = \sqrt{x}$. Also state the domain of f' .

(5 marks)

f) If f and g are both differentiable, show that $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$.

(4 marks)

g) Find $\frac{dy}{dx}$ by implicit differentiation, if $x^2y^2 + x \sin y = 4$.

(4 marks)

h) Find the coordinates of the point on the graph $y = 5x^2 - 3x + 1$ where the gradient is 2.

(3 marks)

QUESTION TWO (20 marks)

a) If $y = u/v$, prove that $\frac{dy}{dx} = \left(\frac{du}{dx}v - u \frac{dv}{dx} \right) / v^2$. (6 marks)

b) Evaluate $\frac{dy}{dx}$ at $x = 2.5$, correct to 4 significant figures, given $y = \frac{2x^2 + 3}{\ln 2x}$. (5 marks)

c) Evaluate $\lim_{x \rightarrow \infty} \frac{4x^4 + 5}{(x^2 - 2)(2x^2 - 1)}$. Give geometrical interpretation of your solution. (5 marks)

d) If $f(x) = \begin{cases} 1, & \text{if } x \leq 3 \\ ax + b, & \text{if } 3 < x < 5 \\ 7, & \text{if } 5 \leq x \end{cases}$

Determine the values of a and b so that $f(x)$ is continuous. (4 marks)

QUESTION THREE (20 marks)

a) Find $D_x f(x)$ given $f(x) = \frac{(x^2 + 1)\cot x}{3 - \cos x \csc x}$. (5 marks)

- b) Using logarithmic differentiation, evaluate $\frac{dy}{dx}$ when $x=1$ given $y = \frac{(x+1)^2 \sqrt{(2x-1)}}{\sqrt{(x+3)^3}}$. (6 marks)
- c) Differentiate $(\ln x)^x + x^{\ln x}$ with respect to x . (5 marks)
- d) Find the derivative of y with respect to θ given $y = (1-\theta)\tanh^{-1} \theta$. (4 marks)

QUESTION FOUR (20 marks)

- a) If $3x^2 + 2x^2y^3 - \frac{5}{4}y^2 = 0$ evaluate $\frac{dy}{dx}$ when $x = \frac{1}{2}$ and $y = 1$. (6 marks)
- b) Find $\frac{dy}{dt}$, given $y = 4\sin(\sqrt{1+\sqrt{t}})$. (5 marks)
- c) If $x = 2t/(t+2)$, $y = 3t/(t+3)$, find $\frac{dy}{dx}$ in terms of t . (4 marks)
- d) Show that the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$ is satisfied when $y = xe^{2x}$. (5 marks)

QUESTION FIVE (20 marks)

- a) The parametric equations for a hyperbola are $x = 2\sec\theta$, $y = 4\tan\theta$. Evaluate $\frac{d^2y}{dx^2}$, correct to 4 significant figures, when $\theta = 1$ radian. (6 marks)
- b) The displacement s cm of the end of a stiff string at time t seconds is given by: $s = ae^{-kt} \sin 2\pi ft$. Determine the velocity and acceleration of the end of the spring after 2 seconds if $a = 3$, $k = 0.75$ and $f = 20$. (5 marks)
- c) Determine for the curve $y = 2x^2 - 3x$ at the point $(2,2)$ the equation of the normal. (4 marks)
- d) The heat capacity c of a gas varies with absolute temperature as shown:
 $c = 26.50 + 7.20 \times 10^{-3}\theta - 1.20 \times 10^{-6}\theta^2$.
 Determine the maximum value of c and the temperature at which it occurs. (5 marks)