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UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR BACHELOR DEGREE

THIRD YEAR FIRST SEMESTER 2022/2023 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: WMB 9303

COURSE TITLE: ORDINARY DIFFERENTIAL EQUATIONS

EXAM VENUE: **STREAM:**

DATE: 13/12/2022 EXAM SESSION: 15.00-17.00PM

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (COMPULSORY) (30 marks)

a) Determine:
   i) the order,
   ii) the degree,
   iii) the unknown function, and
   iv) the independent variable
   for

$$6\left(\frac{d^5p}{dq^5}\right)^6+8\left(\frac{dp}{dq}\right)^9+p^9-p^6=q$$ (4 marks)

b) Find a solution to the boundary-value problem $y''=+4y=0;\ y(0)=1,\ y\left(\frac{\pi}{2}\right)=2,$ if the general solution to the differential equation is known to be $y(x)=C_1\sin 2x+C_2\cos 2x$. (6 marks)

c) Form a suitable differential equation using $y=Axe^{3x}+Be^{3x}$ (5 marks)

d) Find general solution of the differential equation $y'=(y-x-1)^2$ explicitly (4 marks)

e) Verify that the function $x=e^{2t}+e^t$ is a solution of the differential equation $\ddot{x}-4\dot{x}+4x=e^t$ on the interval $(-\infty,\infty)$ (6 marks)

f) According to Newton's Law of cooling, the rate of change of the temperature T satisfies

$$\frac{dT}{dt}=-k(T-T_s)$$

where T_s is the ambient temperature, k is a constant and t is time in minutes. When object is placed in room with the temperature 12^0C, it was found that the temperature of the object drops from 75^0C to 25^0C in 30 minutes. Determine the temperature of an object after 20 minutes. (5 marks)

QUESTION TWO (20 marks)

a) Using substitution of the form $x=x_1+h,\ y=y_1+k,$ reduce the equation below to homogeneous and find its solution

$$y'=\frac{2y-x+5}{2x-y-4}$$ (8 marks)

b) Find y solution of the second order nonlinear equation $y''=-2x(y')^2$ with initial conditions $y(0)=2,\ y'(0)=-1$ (6 marks)

c) Determine whether or not $(3x^2+6xy^2)dx+(6x^2\,y+4y^3)\,dy=0$ is exact. If exact, find the solution. (6 marks)

QUESTION THREE (20 marks)

a) Find the solution of the given differential equation
$y' - 2xy = \cos x - 2x \sin x;\ y(0) = 1$ (6marks)

b) Solve the following Bernoulli differential equation

$x\frac{dy}{dx} + y = y^2 \ln x$ (6marks)

c) Show that
$\left(x^2 + 2xy - y^2\right) + \left(y^2 + 2xy - x^2\right)y' = 0;\ y(1) = -1$ is homogeneous and find its solution.
(7 marks)

QUESTION FOUR (20 marks)

a) Solve the initial-value problem using the method of undetermined coefficients
$y'' - 6y' - 7y = 6e^{2t},\quad y(0) = 5,\quad y'(0) = -3$ (11 marks)

b) Solve the differential equation using the method of variation of parameters
$y'' + y = \sec x$ (9 marks)

QUESTION FIVE (20 marks)

a) Equation $y'' + 9y = 14 \sin 4t$ describes a spring block system that is driven by an oscillatory external for $f(t) = 14 \sin 4t$ in the absence of friction. If the block has an initial position $y(0) = 4$ and an initial velocity $y'(0) = 1$. Find the solution of the initial value problem.
(9 marks)

b) A particle moves vertically under the force of gravity against air resistance Kv^2, where K is a constant. The velocity at any time is given by the differential equation

$$\frac{dv}{dt} = g - Kv^2$$

If the particle starts off from rest, show that

$$v = \frac{\lambda\left(e^{2\lambda kt} - 1\right)}{\left(e^{2\lambda kt} + 1\right)}$$

Such that $\lambda = \sqrt{\frac{g}{K}}$. Then find the velocity as the time approaches infinity. (11 marks)