JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTURIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS

1st YEAR 2nd SEMESTER 2016/2017 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SMA 814

COURSE TITLE: FUNCTIONAL ANALYSIS II

EXAM VENUE: **STREAM: (Msc. Pure Mathematics)**

DATE: **EXAM SESSION: TWO**

TIME: 3.00 HOURS

Instructions:

1. **Answer any THREE questions only**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE [20 MARKS]

a) Define Hilbert Space giving two examples. (5 marks)

b) State and prove the Polarization identity giving possible areas of application. (9 marks)

c) Prove that orthonormal bases for a complex inner product space have the same cardinality. (6 marks)

QUESTION TWO [20 MARKS]

a) State and prove Banach-Steinhouse Theorem (10marks)

b) State and prove Open mapping Theorem. (10 marks)

QUESTION THREE [20 MARKS]

(a) Define convergence and hence distinguish between weak convergence and strong convergence in normed linear space. (5 marks)

(b) Show that if strong limit exists then it is unique. (7 marks)

(c) Prove that strong convergence implies weak convergence but the converse is not necessarily true. (8 marks)

QUESTION FOUR [20 MARKS]

a) Define: Inner product space; Normed space; Banach space. (6 marks)

b) State and prove Cauchy- Buniakowski-Schwarz inequality giving areas of applications in functional analysis. (14 marks)

QUESTION FIVE [20 MARKS]

a) State and prove Riez's representation theorem. (7 marks)

b) Describe Gram-Schmidt orthonormalization process giving real and complex case examples. (6 marks)

c) State and prove Spectral Mapping Theorem. (7 marks)