JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN EDUCATION

4TH YEAR 2ND SEMESTER 2013/2014 ACADEMIC YEAR

CENTRE: MAIN

COURSE CODE: SPH 403

COURSE TITLE: QUANTUM MECHANICS II

EXAM VENUE: AH **STREAM: BSc. .Education**

DATE: 20/12/2013 **EXAM SESSION: 9.00 – 11.00 AM**

TIME: 2 HOURS

Instructions:

1. **Answer question 1(Compulsory) and ANY other 2 questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (Compulsory) (30 Marks)

a. i. State the basic postulates of quantum mechanics, specifying explicitly the roles of the wave function and quantum operators (8 marks)

ii. Show how the quantum operator for the orbital angular momentum can be obtained from two other appropriately defined quantum operators

(2 marks)

b. i. Write down the Pauli spin operators for a spin-$\frac{1}{2}$ particle and derive their algebraic relations (9 marks)

ii. Determine the spin-up and spin –down eigenvalues for a spin-$\frac{1}{2}$ particle (3 marks)

c. Distinguish between the Schroedinger and Heisenberg pictures of quantum mechanics and show that the two pictures are equivalent with respect to the evaluation of expectation value of an operator. (5 marks)

d. Distinguish between bosons and fermions, giving one example of each.

(3 marks)

QUESTION TWO (20 Marks)

a. Obtain the explicit forms of the orbital angular momentum operator components $\hat{L}_x$, $\hat{L}_y$, $\hat{L}_z$ and determine their algebraic relations using the commutation relations for coordinates and linear momentum components (9 marks)

b. i. Write down the expressions for the orbital angular momentum component $\hat{L}_z$ and the square of the total operator $\hat{L}^2$ in spherical polar coordinates. (2 marks)

ii. Show that since $\hat{L}_z$ and $\hat{L}^2$ commute, they have a common quantum mechanical wave function (3 marks)

iii. Obtain an appropriate differential equation for the spherical harmonics satisfying the eigenvalue equations for $\hat{L}_z$ and $\hat{L}^2$. Specify the special functions which are obtained as the general solutions of this differential equation. (6 marks)

QUESTION THREE (20 Marks)

a. Explain the conditions under which a perturbation theory is applicable in both time-dependent and time-independent Schroedinger equations. State one major difficulty usually encountered in a perturbation based solution of each of the time-dependent and time-independent equations (5 marks)

b. i. Write down the time evolution equations for the probability amplitudes of a two-level particle interacting with an external field. (2 marks)

ii. Express the equations in b(i) in matrix form and obtain the general solution if the exrternal field is static (constant). (10 marks)

iii. Calculate the transition probability to the higher level, if the particle was initially in the lower level. (3 marks)

QUESTION FOUR (20 Marks)

a. i. Write down the Hamiltonian for a 2-electron atom such as helium (2 marks)

   ii. Show that the 2-electron atom Hamiltonian can be decomposed into a sum of two 1-electron Hamiltonians characterized by nuclear charge screening. (8 marks)

b. Determine the minimum value of the nuclear charge screening factor if the two electrons are equidistant from the nucleus and use the result to express the corresponding 2-electron atom Hamiltonian as a sum of two independent 1-electron atom Hamiltonians with reduced nuclear charge (10 marks)

QUESTION FIVE (20 Marks)

a. Use a factorization procedure to obtain the general solution of the time-independent Schroedinger equation for a linear harmonic oscillator along the x-axis. (10 marks)

b. Calculate the uncertainty product in the measurements of displacement and linear momentum of the oscillator (10 marks)