JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

4TH YEAR 2ND SEMESTER 2017/2018 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SAC 406

COURSE TITLE: RISK AND CREDIBILITY THEORY

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION 1 [COMPULSORY] [30 Marks]

(a) (i)Explain briefly what is meant by credibility theory, stating the essential features of a risk premium calculated using credibility theory.[**4 Marks**]

(ii)Explain briefly what is meant by the credibility factor. [**4 Marks**]

(b)A random sample $x_1, x_2, ..., x_{20}$ is taken from a distribution having the density function:

$$f(x) = \frac{k}{5}x^{-\frac{4}{5}}/e^{-kx^{\frac{1}{5}}}, \qquad x > 0$$

For this sample $\sum x_i = 247,360$ and $\sum x_i^{\frac{1}{5}} = 102.778$. What is the maximum likelihood estimate of k?

[**8 Marks**]

(c)A random variable X has a continuous uniform distribution on the interval $(0, \theta)$. A statistician wishes to estimate θ on the basis of a single observation from the distribution. The decision function to be used is $d(x) = kx$, and the losses are proportional to the absolute value of the errors. Find the value of k that minimises the risk. [**8 Marks**]

(d)Prove that at least 1082 claims are required for full credibility under the following conditions. Claims amounts have zero variance i.e. the system is based on claim numbers only.We define full credibility to mean that we are 90% confident that our estimate of the Amount A will be within 5% of the unknown true mean. A is compound Poisson which we approximate with the Normal distribution. Assume that, at 95% confidence interval the unit Normal variate = 1.645. [**6 Marks**]

QUESTION 2[20 MARKS]

(a)(i)A random variable S can be expressed as

$$S = Y_1 + Y_2 + ... + Y_N$$

where $Y_i, i = 1, 2, ..., N$ is a sequence of independent and identically distributed random variables each with a mean m and variance s^2, and N, which is independent of this sequence, has the probability distribution function

$$Pr(N = x) = \begin{pmatrix} k + x - 1 \\ x \end{pmatrix} p^k (1 - p)^x : \qquad k > 0, 0 < p < 1$$

Show that

$$E(S) = \left[\frac{k(1 - p)}{p}\right] m$$

and

$$Var(S) = \left[\frac{k(1 - p)}{p^2}\right] (m^2 + ps^2)$$

.

[**8 Marks**]

(ii)For a certain type of insurance policy, losses are assumed to follow a gamma (α, β) distribution with mean Kshs.1600 and variance Kshs$(800)^2$. *Determine* α and β. [**6 Marks**]

(b)Insurance policies providing car insurance are such that the sizes of claims are normally distributed with mean Kshs.1,870 and standard deviation Kshs.610. In one month 50 claims are made. Assuming that claims are independent, calculate the probability that the total of the claim size is more than Kshs. 100,000. **[6 Marks]**

QUESTION 3[20 MARKS]

(a)Define partial and full credibility **[6 Marks]**

(b)The number of claims generated each year by a risk has a Poisson distribution with parameter λ. $\widehat{\lambda}$ is an estimate of λ based on data consisting of the number of claims occurring in each of the past five years. These data are said to be fully credible (k, p) if

$$P[(1-k)\lambda \leq \widehat{\lambda} \leq (1+k\lambda)] \geq p$$

where $k > 0$ and $0 < p < 1$

(i)Find the lowest value of λ for which the data are fully credible (0.075,0.9).

[6 Marks]

(ii)If a total of 384 claims occurred in the past five years and these data are fully credible (k,0.95), find the lowest possible value of k. **[6 Marks]**

QUESTION 4[20 MARKS]

(a)A motor insurer wishes to estimate the claim frequency for a particular risk where claim numbers are assumed to have a Poisson distribution with parameter λ. λ is unknown but is to be regarded as a random variable with a gamma distribution with known parameters α and β. Given that the number of claims in the past n years are $x_1, x_2, ..., x_n$.Show that the Bayesian estimate of λ with respect to quadratic loss function is given by

$$\frac{\alpha + \sum_{i=1}^{n} x_i}{\beta + n}$$

[**8 Marks**]

(b)Explain how the result in (a) may be interpreted in terms of credibility theory, obtaining an expression for the credibility factor. [**6 Marks**]

(c)Hence obtain the Bayesiam estimate of λ when

$$n = 10, \sum_{i=1}^{10} x_i$$

and the gamma prior distribution has mean 1 and standard deviation $\frac{1}{2}$.

[**6 Marks**]

QUESTION 5[20 MARKS]

The data below gives the aggregate claims for residential fire insurance in five successive years experienced by five separate groups of policyholders. You may assume that the claim amounts have been adjusted to remove the effect of inflation and have been measured in some appropriate units.

			Years			
		1	2	3	4	5
Groups	A	103	73	135	91	67
	B	73	138	155	106	133
	C	32	29	21	49	65
	D	102	93	123	111	93
	E	78	104	77	116	118

Calculate the Empirical Bayes Credibility estimates for the coming year for each groups of policyholders.

[20 Marks]