JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR THEDEGREE OF BACHELOR OF EDUCATION (SCIENCE)

4TH YEAR 2ND SEMESTER

MAIN

REGULAR

COURSE CODE: SPH 402

COURSE TITLE: STATISTICAL MECHANICS

EXAM VENUE: **STREAM: (BED SCI)**

DATE: **EXAM SESSION:**

TIME: 2:00HRS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Useful constants

$h = 6.63 \times 10^{-34} Js$

$c = 3 \times 10^{8} m/s$

$k_B = 1.38 \times 10^{-23} JK^{-1}$

QUESTION 1 (30 MARKS)

(a) Define the following terms.

(i) Statistical Physics **(1 mark)**

(ii) Canonical ensemble **(1 mark)**

(iii) Grand canonical ensemble **(1 mark)**

(iv) Phase space **(1 mark)**

(b) The Hamiltonian of a given system in phase space is given by $H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 q^2$

Determine the Hamiltonian canonical equations of motion. **(3 marks)**

(c) Two bosons are in an energy level with degeneracy of 3. Determine the number of microstates of these bosons. **(3 marks)**

(d) State **TWO** features of particles obeying Maxwell-Boltzmann statistics.

(2 marks)

(e) By using Helmholtz free energy, derive expressions for entropy, pressure and chemical potential. **(4 marks)**

(f) Derive the equation for the partition function of a spin-$\frac{1}{2}$ paramagnetic system.

(3 marks)

(g) Explain the formation of Bose-Einstein condensate. **(2 marks)**

(h) Show that the average internal energy in a canonical ensemble is given by

$\overline{E} = \frac{1}{Z}\sum_i \varepsilon_i e^{-\beta\varepsilon_i}$ where each symbol has its usual meaning. **(3 marks)**

(i) Explain the term black body as used in Statistical Physics **(2 marks)**

(j) Determine the percentage error in the use of Stirling's formula to calculate $\ln N!$ for a system of $N = 10000$ particles. **(4 marks)**

Attempt any TWO questions in this section

QUESTION 2 (20 MARKS)

(a) Show that the Maxwell-Boltzmann distribution function is given by

$$\frac{n_i}{g_i} = \frac{N}{Z} e^{-\frac{\varepsilon_i}{k_B T}}$$ **(12 marks)**

(b) Blackbody radiation in a box of volume V and at temperature T has internal energy $U = \sigma V T^4$ and pressure $P = \frac{1}{3}\sigma T^4$, where σ is the Stefan-Boltzmann constant. Determine the entropy and chemical potential in terms of U.

(8 marks)

QUESTION 3 (20 MARKS)

(a) Consider a system of 6 particles that obey Fermi-Dirac statistics with a a degeneracy of 3 in each energy level. Given that the energy levels are equally spaced and the total energy per macrostate is $E = 6\varepsilon$ where $E_1 = \varepsilon$, $E_2 = 2\varepsilon$ and so on, determine the statistical weight of the system. **(10 marks)**

(b) Obtain an expression for the entropy of a thermodynamic system in terms of the statistical weight. **(10 marks)**

QUESTION 4 (20 MARKS)

(a) Show that the energy density in blackbody radiation is given by

$$u(\lambda, T) = \frac{8\pi hc}{\lambda^5\left(e^{\frac{hc}{kT\lambda}} - 1\right)} d\lambda$$ **(15 marks)**

(b) Show that the maximum value of the wavelength for which $u(\lambda, T)$ obtained in 4 (a) is maximum is obtained from $\lambda_m T = 2.9mmK$ **(5 marks)**

QUESTION 5 (20 MARKS)

(a) Derive the average internal energy of a system of N -independent particles in the form $\overline{U} = KT^2\left(\frac{\partial \ln Z}{\partial T}\right)$ **(7 marks)**

(b) Determine the conditions under which a composite isolated system C resulting interaction of two isolated systems A and B attains thermodynamic equilibrium.

(13 marks)