JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION AND ACTUARIAL SCIENCE

4TH YEAR 2ND SEMESTER 2024/2025 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: WMB9402

COURSE TITLE: MEASURE THEORY

EXAM VENUE:AUD **STREAM: BED AND ACT SCIENCE**

DATE: 14/4/25 **EXAM SESSION: 15-17.00 AM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS) [*COMPULSORY*]

a) Let X = {a, b, c}. Determine all the sigma-algebras on X. **[5 mks]**

b) Show that the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$ is measurable. **[5 mks]**

c) Prove that the Lebesgue measure *m* of the interval [a, b] is equal to b - a. **[5 mks]**

d) State without proof the Egorov's Theorem giving one example of its applications. **[5 mks]**

e) An outer measure is translation invariant. Describe this property giving its significance in measure theory. **[5 mks]**

f) Evaluate the Lebesgue integral over a simply connected curve C of the function $f=z-z^2$ with respect to z where C is the upper half of the circle $|z|=1$. **[5 mks]**

QUESTION TWO [20 MARKS]

a) Define the Lebesgue measure on the real line R and show that the empty set has measure zero. **[8 mks]**

b) Let A_n be a sequence of disjoint measurable sets. Show that $\mu(\cup A_n) = \Sigma \mu(A_n)$, where μ is the Lebesgue measure. **[12 mks]**

QUESTION THREE [20 MARKS]

a) Define the Lebesgue integral of a simple function outlining its key properties. **[9 mks]**

b) Let μ be a sigma measure on a measurable space (X, A), and let f: X → R be a Lebesgue measurable function. Prove that if $f \geq 0$, then the Lebesgue integral of f is given by $\int_X f \, d\mu = \sup\{\int_X \varphi \, d\mu \mid 0 \leq \varphi \leq f$ and φ is simple }. **[11 mks]**

QUESTION FOUR [20 MARKS]

a) State and prove Fatou's Lemma. **[10 mks]**

b) Compare and contrast Lebesgue integral and Riemann integral **[10 mks]**

QUESTION FIVE [20 MARKS]

a) State and prove the Monotone Convergence Theorem. **[10 mks]**

b) Describe any five applications of the Monotone Convergence Theorem **[10 mks]**