JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES
UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OFSCIENCE IN EDUCATION (SCIENCE AND ART)
2nd YEAR 1st SEMESTER 2022/2023 ACADEMIC YEAR
MAIN REGULAR

COURSE CODE: WMB 9210

COURSE TITLE: PROBABILITY DISTRIBUTION THEORY II

EXAM VENUE: **STREAM: (BSc. Actuarial Science)**

DATE: 15/12/2022 **EXAM SESSION:9.00-11/00AM**

TIME: 2.00 HOURS

Instructions:

i. Answer questions one and any other two.
ii. Candidates are advised not to write on the question paper.
iii. Candidates must hand in their answer booklets to the invigilator while in the examination room.

QUESTION ONE (30 marks)

a) If $x_1$and x_2 are two independent random variables with mean μ_1 and $\mu_2$with the following distribution.

$$f(x_1, x_2) = \frac{\mu_1^{x_1}\mu_2^{x_2}e^{-\mu_1-\mu_2}}{x_1!x_2!}$$

i. Find the moment generating function of $y_1 = x_1 + x_2$ (6 marks)

ii. What type of distribution is this? (2 marks)

iii. What is the mean of the distribution (2 marks)

b) Suppose that x is a continuous random variable with the following pdf

$$f(x, y) = \begin{cases} k(2xy + x), & 0 < x < 1;\ 0 < y < 2 \\ 0, & elsewhere \end{cases}$$

Let y=2x+1

i. Determine the value of the constant k (3 marks)

ii. Determine the marginal density function and check whether xand y are independent (4 marks)

c) Suppose the probability of a success outcome in an experiment is 0.4. in the experiment, 15 independent trials of the experiment were made. If x is the number of successes, determine

i. $Pr(x=3)$ (3 marks)

ii. $Pr(6 \leq x \leq 9)$ (3 marks)

iii. $Pr(x \geq 10)$ (3 marks)

d) If x is a random variable such that $E(x)=3$, $E(x^2)=13$. Use the Chebyshev's inequality to determine the lower bound for $Pr(-2<x<8)$. (4 marks)

QUESTION TWO (20mks)

Suppose X and Y are jointly distributed random variableswith the following pdf

$$f(x, y) = \begin{cases} k(2xy + y + 1), & 0 < y < 1, \quad 0 < x < 2 \\ 0, & elsewhere \end{cases}$$

i) Determine the value of the constant k (5 marks)

ii) Compute $f(x|y)$ (5 marks)

iii) Determine $E(x|y)$ (5 marks)

iv) Determine the $var(x|y)$ (5 marks)

QUESTION THREE (20mks)

Use the joint distribution function below to answer the following questions

$$f(x, y) = \begin{cases} \frac{1}{2}xy, & 0 < y < 1, \quad 0 < x < 2 \\ 0, & elsewhere \end{cases}$$

i. E(x) (3 marks)

ii. E(y) (3 marks)

iii. E(xy) (4 marks)

iv. Cov(xy) (4 marks)

v. Find regression equation of Y on X (6 marks)

QUESTION FOUR (20mks)

a) Each student of CEES-UON was classified according to his year of study and the number of times he visited the national museum. The proportion of students in each classification is given by

Study year	**Never**	**Once**	**More than once**
1st year	0.08	0.10	0.04
2nd year	0.04	0.10	0.04
3rd year	0.04	0.20	0.09
4th year	0.02	0.15	0.10

i. What is the probability that the student never visited the museum? (2 marks)

ii. What is the probability the student is in 3rd year of study? (2 marks)

iii. Suppose a student selected at random is a 3rd year, what is the probability that he or she never visited the museum? (5 marks)

iv. Suppose a student who is chosen has visited the museum 3 times, what is the probability that he or she is a 4th year (5 marks)

b) Consider a sample of size 2 drawn without replacement from an urn containing three balls numbered 1,2 and 3. Let x be the number of the first ball drawn and y be the larger of the two numbers drawn.

i. Find the joint discrete probability density of x and y. (3 marks)

ii. Find the $Pr(x=1|y=3)$ (3 marks)

QUESTION FIVE (20mks)

Ten students were subjected to tests x and y and the results recorded as per the below table

Student	A	B	C	D	E	F	G	H	I	J
Test x results	13	30	15	9	17	22	8	17	14	12
Test y results	21	12	19	22	18	13	25	15	20	16

i. Determine the coefficient of correlation between the two sets of results and use the answer obtained to comment on the relative performance on the two tests. (6 marks)

ii. An eleventh student came late and it was found that she had done test y. also, she attempted test x and obtained 23 marks. Determine the best estimate of the marks she could have obtained if he had attempted test y. (14 marks)