SMA 403 TOPOLOGY I
Answer QUESTION ONE and any other TWO questions

QUESTION ONE(30MARKS)

(a) Define the following terms.
(i)Limit point of a subset E of X.
(ii) An interior point of a set E.

(4MARKS)

(b) Let $X = \{a, b, c, d, e\}$ and $\tau = \{\emptyset, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}, X\}$ be a topology on X. Let $E = \{a, d, e\}$. Determine;
(i) The derived set of E.

(4MARKS)

(ii) The interior of E.

(3MARKS)

(iii) Prove that every T_1 space is hereditary.

(4MARKS)

(c)(i) Define what is meant by a function

$$f : (X, \tau) \to (Y, \tau^*)$$

is continuous at $x \in X$.

(2MARKS)

(ii) Prove that a constant function is continuous relative to any topology.

(3MARKS)

(d)(i) Define a homeomorphism.

(2MARKS)

(ii) Using the definition in (i) above, show that $[0, 1]$ is homeomorphic to $[a, b]$.

(5MARKS)

(e)(i) State the T_o axiom.

(2MARKS)

(ii) Prove that every subspace of a T_2 space is a T_2 space.

(5MARKS)

QUESTION TWO(20MARKS)

(a)(i) Prove that if $A \subset B$ then $A' \subset B'$

(4MARKS)

(ii) Prove that a composition of two continuous functions is continuous.

(6MARKS)

(b) Prove that every metric space is a T_3-space.

(6MARKS)

QUESTION THREE(20MARKS)

(a)(i)State the T_1 axiom.

(2MARKS)

(ii) Prove that X is a T_1 space if and only if every singleton subset of X is closed.

(8MARKS)

(b) Let $X = \{a, b, c, \}$ and $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ be a topology on X.
Determine whether or not ;
(i) (X, τ) is a regular space.

(2MARKS)

(ii) (X, τ) is a T_2 space.

(2MARKS)

(iii) What conclusion would you make from the computation of (i) and (ii) above.

(1MARKS)

(c) Let $f : X \to Y$ be a function such that $f^{-1} : Y \to X$ exist. Let $E \subset Y$ and define $f^{-1} = \{x \in X : f(x) \in E\}$. Using this definition prove that $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$.

(5MARKS)

QUESTION FOUR(20MARKS)

(a)Prove that a function $f : X \to Y$ is continuous if for every subset A of X $f(\overline{A}) \subset \overline{(f(A))}$.

(9MARKS)

(b)(i) Define a local base of a topological space (X, τ).

(3MARKS)

(ii) Show that if $X = \{a, b, c\}$ then $\beta = \{\{a\}, \{b\}, \{c\}\}$ forms a basis for discrete topology.

(3MARKS)

(c)(i) Define a cover for a space X

(2MARKS)

(ii) Construct a cover for X which is not an open cover if $X = \{a, b, c, d\}$

(2MARKS)

QUESTION FIVE(20MARKS)

(a)(i) State the Tietze's characterization of normality.

(3MARKS)

(ii) Prove that every metric space is normal.

(6MARKS)

(b) Let $X = \{a, b, c, d, e\}$ and $\tau = \{\emptyset, \{a\}, \{e\}, \{a, b\}, \{a, b, e\}, X\}$ be a topology on X. Determine;
(i)$Ext(A)$ if $A = \{b, c, e\}$

(2MARKS)

(ii) $Bound(A)$ given $A = \{b, c, e\}$

(4MARKS)

(iii) If $\beta = \{\{a, b\}, \{b, c\}\}$ forms a basis for the topology.

(1MARKS)

(c) Write short notes on product topology.

(4MARKS)