JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

SPECIAL RESIT 2015/2016 ACADEMIC YEAR

MAIN RESIT

COURSE CODE: SAS 102

COURSE TITLE: PROBABILITY AND DISTRIBUTION THEORY 1

EXAM VENUE: LAB 1 **STREAM: (BSc. Actuarial)**

DATE: 05/05/2016 **EXAM SESSION: 9.00 – 11.00 AM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (COMPULSORY)-(30 MARKS)

a) The joint density function of two continuous random variables X and Y is given by

$$f(x,y)=\begin{cases}k(2x-y), & 0\le x\le 2\ ,0\le y\le 3\\ 0, & otherwise\end{cases}$$

Obtain

i. the value of k.

ii. the expected value of X (8 marks)

b) Let $f(x,y)=\begin{cases}6x^2y, & 0\le x\le 1\ ,0\le y\le 1\\ 0, & otherwise\end{cases}$

be the p.d.f. of two random variables *X* and *Y*, which must be of continuous type. Find $P(0<x<{}^{3}/_{4}, y>{}^{1}/_{3})$ (8 marks)

c) The joint probability function for two discrete random variables X and Y is tabulated as shown

	Y=0	Y=1	Y=2	Y=3
X=1	0.06	0.02	0.04	0.08
X=2	0.15	0.05	0.10	0.20
X=3	0.09	0.03	0.06	0.12

Determine

i. Marginal distributions of X and Y . (3 marks)

ii. P(X≤2, Y≥2) (3 marks)

d) Given $f(x,y)=\begin{cases}2e^{-x-2y}, & 0<x<\infty\ ,0<y<\infty\\ 0, & otherwise\end{cases}$.

Determine

i. $P(X>1,Y<1)$

ii. $P(X<Y=10)$ (8 marks)

QUESTION TWO (20 MARKS)

a) A random variable X has the Beta distribution with parameters α and β as shown below.

$$f(x)=\begin{cases}\frac{\Gamma(\alpha+\beta)}{\Gamma\alpha\Gamma\beta}x^{\alpha-1}(1-x)^{\beta-1}, & 0<x<1\ ,\alpha>0,\beta>0\\ 0, & otherwise\end{cases}$$

Determine by derivation for this distribution, the standard deviation when $\alpha=8,\beta=10$.

. (10 marks)

b) Determine the value of c for which the function below is a joint probability density function hence work out f(y).

$$f(x,y)=\begin{cases}c(x+y), & 0<x<3\ ,x<y<2x+1\\ 0, & otherwise\end{cases}$$

(10 marks)

QUESTION THREE (20 MARKS)

a) The joint probability function of two discrete random variables X and Y is given by

$$f(x,y)=\begin{cases}k(2x+y), & 0\le x\le 3\ ,1\le y\le 3\\ 0, & otherwise\end{cases}$$

Obtain

i. the value of k.

ii. deduce whether or not X and Y independent? (10 marks)

b) Consider the Weibull distribution with parameters a and b

$$f(x)=\begin{cases}abx^{b-1}e^{-ax^b}, x>0\\ 0,\ otherwise\end{cases}$$

Obtain a general expression for the mean and the third raw moment for the distribution.

. (10 marks)

QUESTION FOUR (20 MARKS)

a) The joint p.d.f of three continuous random variables X , Y and Z is defined as follows

$$f(x,y,z)=\begin{cases}k(xy+z), & 0<x<3\ ,0<y<4,0<z<1\\ 0, & otherwise\end{cases}$$

Calculate:

i. the value of k ,

ii. the marginal distribution of X

iii. $E(YZ/X=2)$ (20 marks)

QUESTION FIVE (20 MARKS)

a) Let $f(x,y)=\begin{cases}cx^2y,\ 0<x<2\ ;0<y<1,\\ 0,\ otherwise\end{cases}$.

Find C hence deduce the relationship between $E(2X-3Y)$ and $2E(X)-3E(Y)$

. (20 marks)