



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE
UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE
ACTUARIAL EXAMINATION

SPECIAL RESIT 2020/2021 ACADEMIC YEAR
REGULAR (MAIN)

COURSE CODE: SMA414

COURSE TITLE: FOURIER ANALYSIS

EXAM VENUE:

STREAM: (BSc/BED)

DATE.....

EXAM SESSION:

TIME: 2.00 HOURS

Instructions:

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question.1 [30 marks] Compulsory

(a) Assume that $f(x)$ has a uniformly convergent Fourier series

$$f(x) \approx \frac{a_0}{2} + \sum_{n=1}^{\infty} \{a_n \cos(nx) + b_n \sin(nx)\}, \quad -\pi \leq x \leq \pi.$$

Prove that

(i) $\int_{-\pi}^{\pi} f(x) dx = \frac{\pi a_0}{2}$

(ii) $a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx$

(iii) $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx$

iv) $\frac{1}{\pi} \int_{-\pi}^{\pi} (f(x))^2 dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} \{a_n^2 + b_n^2\}$ [11 marks]

(b) Given the function $f(x) = 4x$, $-50 < x < 50$ $f(x) = f(x+100)$

(i) Sketch graph of $f(x)$ over the interval $-400 < x < 400$

(ii) State period of $f(x)$ [9 marks]

(c) For the step function

$$f(x) = \begin{cases} 2\pi & ; 0 \leq x < \pi \\ -2\pi & ; -\pi \leq x < 0 \end{cases}$$

deduce that $f(x)$ may be expressed in an infinite series

$$f(x) = \frac{1}{\pi} \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{4}{n} \sin nx$$
 [10marks]

Question2 [20 marks]

(a) Assume that $f(x)$ has a uniformly convergent Fourier series

$$f(x) \approx \frac{a_0}{2} + \sum_{n=1}^{\infty} \{a_n \cos(nx) + b_n \sin(nx)\}, \quad -\pi \leq x \leq \pi.$$

Discuss the Gibb's Phenomenon of $f(x)$. [8 marks]

b) One cycle of a periodic waveform $y = f(x)$ of period 2π is defined by the below data.

x^0	0	30	60	90	120	150	180	210	240	270	300	330
$y(x)$	15	20	23	24	20	8	3	4	9	12	10	11

Determine the approximate Fourier series for $y = f(x)$ up to and including the third harmonic. [12 marks]

Question3 [20 marks]

Given real valued function $y = f(x)$ for which

$$f(x) = \begin{cases} 0 & -\pi < x < \pi \\ -x & 0 < x < \pi \end{cases}$$

$$f(x) = f(x + 2\pi)$$

(a) Sketch the graph of $f(x)$ over the interval $-4\pi < x < 4\pi$ [5 marks]

(b) State period of $f(x)$ [3 marks]

(c) Show that

$$f(x) = \frac{\pi}{4} + \frac{2}{\pi} \left\{ \cos x + \frac{1}{9} \cos 3x + \frac{1}{25} \cos 5x + \frac{1}{49} \cos 7x \dots \right\} + \sin x - \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x - \frac{1}{4} \sin 4x \dots$$

[12 marks]

Question4 [20 marks]

The centre of an elastic string 4m long is deflected initially to a point P 0.01m high and then released. The initial velocity is zero. Given u the wave displacement of the string satisfies the equation

$$u_{xx} = \frac{1}{100} u_{tt}, \quad u = u(x, t) \text{ subject to boundary and initial conditions}$$

$$u(x, 0) = \begin{cases} \frac{x}{200} & 0 < x < 2 \\ \frac{2}{100} - \frac{x}{200} & 2 < x < 4 \end{cases}$$

$u_t(x, 0) = 0 ; 0 < x < 4, u(0, t) = u(4, t) = 0 ; 0 < t$, determine the wave movement of the string.

[20 marks]

Question5 [20 marks]

(a) Consider the function defined on $[0, \pi)$ by

$$f(x) = \begin{cases} 2x & 0 \leq x < \frac{\pi}{2} \\ \pi - 2x & \frac{\pi}{2} \leq x < \pi \end{cases}$$

Obtain the Fourier **half-range** cosine coefficients for this function $f(x)$.

Sketch the symmetric even periodic extension of $f(x)$ on $(-\pi, \pi)$

$$f(x) = 10, \quad 0 < x < \pi$$

[10 marks]

(b) Let $f(x)$ be periodic function with period 2π with
 $f(x) = |\sin 2x|, -\pi \leq x \leq \pi$

(i) State giving reasons whether $f(x)$ is odd or even.

(ii) Sketch $f(x)$ over the interval $[-4\pi, 4\pi]$

(iii) Obtain the Fourier series expansion for $f(x)$

[10 marks]

