



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE

ACTUARIAL

2ND YEAR 1ST SEMESTER 2015/2016 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SAC 201

COURSE TITLE: FINANCIAL MATHEMATICS I

EXAM VENUE: LAB 1

STREAM: BSc. Actuarial Sc.

DATE: 18/04/16

EXAM SESSION: 9.00 – 11.00 am

TIME: 2.00 HOURS

Instructions:

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**
- 4. Symbols used have their usual meaning in actuarial science**

QUESTION 1 [COMPULSORY] [30 Marks]

(a) Find the effective annual rate of interest corresponding to:

(i) a nominal rate of 11% p.a convertible half-yearly [2 Marks]

(ii) a nominal rate of interest of 12% p.a convertible monthly [2 Marks]

(iii) a rate of discount of 10% p.a on a 91-day government bill. [2 Marks]

(b) Write down the simple formulae expressing:

(i) v in terms of δ [1 Mark]

(ii) i in terms of δ [1 Mark]

(iii) d in terms of i [1 Mark]

where i is rate of interest, d is the rate of discount, δ is the force of interest and v is the discounting factor.

(c)(i) Define annuity-certain and annuity-due stating the formula for each.

[4 Marks]

(ii) Let X denote the present value of an annuity consisting of payments of Kshs.2,000 payable at the end of each of the next 8 years, valued using an interest rate of 8% p.a convertible quarterly and let Y denote the present value of an annuity consisting of payments of Kshs.4,000 payable at the end of every fourth year for the next 16 years, valued using an interest rate of 8% p.a convertible half-yearly. Calculate the ratio $\frac{X}{Y}$ [7 Marks]

(d) In return for an immediate outlay of Kshs.10,000 an investor will receive Kshs.6,000 in one year's time and Kshs.6,600 in two year's time. Find to the nearest 0.1% the internal rate of return for this investment

[5 Marks]

(e) A person who has no spare cash can make the investment described in (d) above by borrowing the initial outlay from a bank. If the bank loans are granted on the basis of an interest rate of 16% p.a and may be partially repaid at any time, should the person make the investment and if so, what profit will he have made at the end of the completed transaction?

[5 Marks]

QUESTION 2 [20 Marks]

(a) Define the accumulation factor $A(t, t + h)$ and give a formula for the force of interest $\delta(t)$ per unit time in terms of the accumulation factor.

[6 Marks]

(b) The force of interest $\delta(t)$ at time t (measured in years) is given by $\delta(t) = 0.001t + 0.04$

Calculate the corresponding nominal rate of interest for the period $t = 1$ to $t = 2$

[5 Marks]

(c) Calculate the accumulated value of an investment of Kshs.100 for 6 months at the following rates of interest

(i) a force of interest of 0.05 p.a [3Marks]

(ii) a rate of interest of 5% p.a convertible monthly [3 Marks]

(iii) an effective rate of interest of 5% p.a [3 Marks]

QUESTION 3 [20 Marks]

The force of interest $\delta(t)$ is a function of time, and at any time t , measured in years, is given by the formular:

$$\delta(t) = \begin{cases} 0.04 + 0.005t & 0 \leq t < 6 \\ 0.16 - 0.015t & 6 \leq t < 8 \\ 0.04 & 8 \leq t \end{cases}$$

- (a) Derive the expression in terms of t for the accumulated amount at time t of an investment of 1 at time 0 [6 Marks]
- (b) Calculate the value at time 0 of Kshs.100 due in time 8 [6 Marks]
- (c) Calculate the accumulated value at time 10 of a payment stream, paid continuously from $t = 6$ to $t = 8$, under which the rate of payment at time t is $\rho(t) = 16 - 1.5t$ [8 Marks]

QUESTION 4 [20 Marks]

An investor is considering two projects A and B. Project A involves the investment of Kshs.1 million at the outset. The only income to be received will be a payment of Kshs.3.5 million after ten years. Project B also involves the investment of Kshs. 1 million at the outset. Income will be received from this project continuously. In the first year the rate of payment will be Kshs.0.08 million, in the second year Kshs.0.09 million, in the third year Kshs.0.10 million each year thereafter until the tenth year, after the end of which no further income will be received.

- (a) Calculate the net present value of both investment projects at a rate of interest of 4% per annum. [8 Marks]
- (b) Show that the discounted payback period for project A is after that of project B. [8 Marks]
- (c) Explain which project is most desirable. [4 Marks]

QUESTION 5 [20 Marks]

A company is considering two capital investments projects. Project A requires an immediate expenditure of Kshs.1,000,000 and will return Kshs.270,000 at the end of each of the next eight years. Project B requires an immediate investment of Kshs.1,200,000 together with further expenditure of Kshs.20,000 at the end of each of the first three years, and will produce returns of Kshs.1,350,000 at the end of each of the sixth, seventh and eighth years.

- (a) Calculate the internal rate of return per annum for each project [12 Marks]
- (b) Find the net present value of each project on the basis of an effective annual interest rate of 15% [8 Marks].

Comment briefly on your answers.