JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICAL & ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR THE BACHELORS DEGREE

1ST YEAR 1ST SEMESTER 2013/2014 ACADEMIC YEAR

CENTRE: MAIN

COURSE CODE: SMA 210

COURSE TITLE: PROBABILITY AND DISTRIBUTION THEORY I

EXAM VENUE: LAB 3 **STREAM: (Bed , B Sc and Special Eduction)**

DATE: 23/4/2014 **EXAM SESSION: 2.00 – 4.00 PM**

TIME: 2 HOURS

Instructions:

1. Answer question 1 (Compulsory)and ANY other 2 questions
2. Candidates are advised not to write on the question paper.
3. Candidates must hand in their answer booklets to the invigilator while in the examination room.

JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
YEAR TWO SEMESTER ONE EXAMINATIONS
SMA 210: PROBABILITY AND DISTRIBUTION THEORY 1
APRIL 2014
BED. SCIENCE, BED. ARTS AND SPECIAL NEEDS EDUCATION

TIME 2 HOURS

INSTRUCTIONS:

1. Answer question ONE(compulsory) and ANY other TWO questions in this paper
2. Answer all the questions in the answer booklet provided.
3. Do not write on this paper.

QUESTION ONE (COMPULSORY) – (30 MARKS)

a) The joint probability function for two discrete random variables X and Y is tabulated as shown

	Y=0	Y=1	Y=2	Y=3
X=1	0.06	0.02	0.04	0.08
X=2	0.15	0.05	0.10	0.20
X=3	0.09	0.03	0.06	0.12

Determine:

i. Whether or not the variables are independent. (5marks)
ii. P(X=x, Y≥2) (3 marks)

b) Suppose we know that $\theta = 15$, find the difference between the mean and fourth raw moment for the exponential distribution with parameter θ given as follows

$$f(x,\theta) = \begin{cases} \theta e^{-\theta x}, x > 0, \theta > 0 \\ 0, otherwise \end{cases}$$ (7marks)

c) The joint p.d.f of three continuous random variables X , Y and Z is defined as follows

$$f(x,y) = \begin{cases} k(xy+z), & 0 < x < 3, 0 < y < 4, 0 < z < 1 \\ 0, & otherwise \end{cases}$$

Calculate:

i. the value of k , (4marks)
ii. the marginal distribution of X (3marks)
iii. P(X≥2) (2marks)

d) A random variable X has the Beta distribution with parameters α and β as shown below.

$$f(x) = \begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma\alpha\Gamma\beta} x^{\alpha-1}(1-x)^{\beta-1}, \; 0 < x < 1, \alpha > 0, \beta > 0 \\ 0, \; otherwise \end{cases}$$

Determine $E(X^3)$ for this distribution when; $\alpha = 8, \beta = 14$, (6marks)

QUESTION TWO (20 MARKS) Type equation here.

a) Derive the mean and variance for the gamma distribution

$$f(x) = \begin{cases} \frac{x^{\alpha-1} e^{-x/\beta}}{\beta^{\alpha}\Gamma(\alpha)}, & x > 0, \alpha > 0, \beta > 0 \\ 0, & x \le 0 \end{cases},$$

with scale parameter β and shape parameter α hence by stating clearly how the Chi-square random variable with V degrees of freedom may be regarded as Gamma, deduce the variance for the Chi-square distribution.
(12marks)

b) Determine the value of c for which the function below is a joint probability density function hence give the marginal distribution of X and Y.

$$f(x,y) = \begin{cases} c(x+y), & 0 < x < 3, x < y < 2x+1 \\ 0, & otherwise \end{cases}$$

(8marks)

QUESTION THREE (20 MARKS)

a) The joint probability function of two discrete random variables X and Y is given by $f(x,y) = \begin{cases} k(2x-y), & 0 \le x \le 2, 0 \le y \le 3 \\ 0, & otherwise \end{cases}$

Obtain

i. the value of k.

ii. the variance covariance matrix and hence deduce whether or not X and Y are independent? (10marks)

b) Consider the Weibull distribution with parameters a and b

$$f(x) = \begin{cases} abx^{b-1}e^{-ax^b}, & x > 0 \\ 0, & otherwise \end{cases}$$

Obtain a general expression for the mean and variance of X
(10marks)

QUESTION FOUR (20 MARKS)

a) Suppose X and Y are two discrete random variables whose joint p.m.f is tabulated as follows

	Y=0	Y=1	Y=2	f(x)
X=0	0.10	0.10	0.20	0.40
X=1	0.0	0.15	0.05	0.20
X=3	0.10	0.20	0.10	0.40
f(y)	0.20	0.45	0.35	1

Use the table to:

i. Obtain $P(X = Y)$ (2marks)

ii. Obtain $P(X \ge 1, Y \ge 1)$ (2marks)

iii. compute $cov(X,Y)$. (6marks)

b) Let X and Y be two independent standard normal random variables. Let $U = {}^{X}/_{Y}$ and $V = X + Y$ be two new random variables. Determine the joint p.d.f of U and V.

(10marks)

QUESTION FIVE (20 MARKS)

Let $f(x, y) = \begin{cases} cx^2y, & 0 < x < 2\,; 0 < y < 1, \\ 0, & otherwise \end{cases}$.

i. Show that $E(X + 2Y) = E(X) + 2E(Y)$

(8marks)

ii. Determine: $E(X/Y)$, $Var\ (X/Y)$, $Var\ (Y/X)$ (12marks)