JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

SPECIAL RESIT 2018/2019 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SAS 303

COURSE TITLE: THEORY OF ESTIMATION

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: EXAM SESSION:

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (20 MARKS)

a) Define clearly the following terms as used in estimation theory.
   i. Consistency
   ii. Completeness
   iii. Unbiasedness
   iv. Sufficiency (8marks)

b) Let $X_1, X_2, \dots, X_n$ be iid Poisson random variables with $f(x_i, \theta) = \frac{e^{-\theta}\theta^{x_i}}{x_i!}$ $x = 0,12, \dots$ Show that $\bar{x}$ is consistent for θ (8 marks)

c) Let $X_1, X_2, \dots, X_n$ be iid $N(\mu, \sigma^2)$. Find a joint sufficient statistic for $\theta = (\mu, \sigma^2)$ (8marks)

d) Let $X_1, X_2, \dots, X_n$ be a random sample of n observations from a population with p.d.f $f(x) = \theta x^{\theta - 1}$ $, 0 < x < 1$. Find the Maximum Likelihood estimator of θ. (6 marks)

QUESTION TWO (20 MARKS)

a) Let $X_1, X_2, \dots, X_n$ be a random sample of n observations from a population having p.d.f $f(x) = \begin{cases} \frac{2}{\theta^2}(\theta - x) & , 0 \le x \le \theta \\ 0, & otherwise \end{cases}$

   Obtain the mean of this distribution hence show that $3\bar{x}$ is consistent for θ (10marks)

b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from $N(\mu, \sigma^2)$. Find the Cramer Rao lower bound for estimation of the square of the mean. (10marks)

QUESTION THREE (20 MARKS)

Find the estimator of α and β in

$$, \qquad f(x) = \begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma\alpha\Gamma\beta} x^{\alpha-1}(1-x)^{\beta-1}, & 0 < x < 1, \alpha > 0, \beta > 0 \\ 0, & otherwise \end{cases}$$

By method of moment. (20 marks)

QUESTION FOUR (20 MARKS)

a) Suppose we have a random sample of size 2n from a population denoted by X where$E(X) = \mu$, Var (X) = σ^2. Let $\bar{X}_1 = \frac{1}{2n}\sum_{i=1}^{2n}(X_i)$, $\bar{X}_2 = \frac{1}{n}\sum_{i=1}^{n}(X_i)$ be two estimators of μ. Investigate $\bar{X}_1$ and $\bar{X}_2$ for unbiasedness and consistency. Which between the two is the better estimator of μ. (12 marks)

b) Use the Lehmann Scheffe method of construction of minimal sufficient statistics to find the minimal sufficient statistic for $\theta = (\mu, \sigma^2)$ given $X_1, X_2, \ldots, X_n$ are iid random variables from $N(\mu, \sigma^2)$ (8 marks)

QUESTION FIVE (20 MARKS)

a) Let $X_1, X_2, \ldots, X_n$ be a random sample from some population with finite mean μ and finite variance σ^2. Show that the sample variance $S^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2$ is biased for the population variance. (10marks)

b) Let $X_1, X_2, \ldots, X_n$ be a random sample of n observations from a population having p.d.f $f(x) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} & , 0 \leq x \leq \infty \\ 0, & otherwise \end{cases}$

Find unbiased estimators of θ and θ^2 (10 marks)