



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

**SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL
SCIENCES**

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE

ACTUARIAL SCIENCE

3RD YEAR 2ND SEMESTER 2024/2025

REGULAR (MAIN)

COURSE CODE: WAB 2304

COURSE TITLE: METHODS OF ACTUARIAL INVESTIGATIONS II

EXAM VENUE: LAB 4

STREAM: (BSc Actuarial Science)

DATE: 14/4/25

EXAM SESSION: 15-17.00 HRS

TIME: 2.00 HOURS

Instructions:

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

- a. Explain why it is necessary to graduate crude rates of mortality for practical use. (4 marks)
- b. For a given set of data, crude estimates $\{\hat{\mu}\}$ for ages $x = 30, 31, \dots, 79$ have been calculated. These rates have been graduated assuming that the underlying force of mortality follows Makeham's law $\mu_x = A + Bc^x$. The graduated rates are now being tested for adherence to data. Give a formula for the test statistic and explain how the test is carried out. (4 marks)
- c. Fill the table below with the respective estimates

Definition of x	Rate interval	$\hat{\mu}$ estimates	\hat{q} estimates
Age last birthday	$[x, x+1]$		
Age nearest birthday	$[x - 1/2, x + 1/2]$		
Age next birthday	$[x-1, x]$		

- (5 marks)
- d. Outline the differences between the two-state model and the Poisson model when used to estimate transition rates (4 marks)
- e. A survival model for an elderly population has two states A and D, representing alive and dead. The force of mortality at age t years $\mu(t)$, i.e the transition rate from A to D, is given by:

$$\mu(t) = 0.0001 \times (1.10)^t$$
 Calculate the probability that a 60-year-old will survive to age 80. (3 marks)
- f. A certain species of small mammal is subject to a constant force of mortality of 0.2 pa over the first year of life. Calculate the expected number of deaths in the first year of life from a population of 1,000 new births. (3 marks)
- g. In a mortality investigation, females aged 65 last birthday were observed. The following data values were recorded:
 Total waiting time = 916 years
 Observed number of deaths = 10
 Calculate an approximate 95% confidence interval for the force of mortality of females aged 65 last birthday, assuming that the force is constant over this year of age. (4 marks)
- h. State three assumptions of two-state Markov model.

(3 marks)

QUESTION TWO (20 MARKS)

- a. The disreputable insurance company had mixed fortunes in the year 2018. At both the start and the end of the year 547 policies were in force in respect of policyholders aged 40 last birthday, but these figures do not tell the whole story.

There was adverse publicity early in the year linking the company's investment managers with a gambling syndicate. As a result, many policyholders 'took their money elsewhere'. Following a successful marketing campaign offering a free toaster to all applicants, the number of policyholders aged 40 last birthday rose from 325 at 1 June 2018 to 613 at 1 September 2018.

Calculate an approximate value for the central exposed to risk at age 40 last birthday for the calendar year 2018.

(10 marks)

- b. A mortality investigation was held between 1 January 2016 and 1 January 2018. The following information was collected. The figures in the table below are the numbers of lives on each census date with the specified age labels.

	DATE		
Age last birthday	1.1.16	1.1.17	1.1.18
48	3,486	3,384	3,420
49	3,450	3,507	3,435
50	3,510	3,595	3,540

During the investigation there were 42 deaths at age 49 nearest birthday. Estimate μ_{49} stating any assumptions that you make.

(10 marks)

QUESTION THREE (20 MARKS)

- a. Two graduations have been carried out and you have obtained the following results:
Graduation 1 covers 40 age groups and has produced 13 positive and 27 negative standardised deviations.
Graduation 2 covers 7 age groups and has produced 5 positive and 2 negative standardised deviations.

Carry out the signs test on these graduations, clearly stating the probability value and your conclusion in each case.

(10 marks)

- b. Carry out the smoothness test on the following set of graduated rates:

x	$\hat{\mu}_x$
55	0.00429
56	0.00478

57	0.00535
58	0.00596
59	0.00667
60	0.00754
61	0.00867

(10 marks)

QUESTION FOUR (20 MARKS)

- a. Consider a cohort of people currently aged exactly 65, who in the absence of the new cure were originally expected to follow the mortality of the ELT15 (Males) table over their next two years of age. Recalculate the entries for d_x , l_x and q_x in the below life table, allowing for the changes in death rates from this particular cancer that have been predicted for the next two years.

Age x	l_x	d_x	q_x
65	79,293	1,940	0.02447
66	77,353	2,097	0.02711
67	75,256		

(10 marks)

- b. i.) Explain the notation and meaning of the parameters α_x and $f_{n,x}$ in the following reduction factor formula:

$$R_{x,t} = \alpha_x + (1 - \alpha_x)(1 - f_{n,x})^{t/n}$$

(4 marks)

- ii.) The mortality rate for the base year of a mortality projection has been estimated to be:

$$m_{60,0} = 0.006$$

It is believed that the minimum possible mortality rate for lives aged 60 is 0.0012. It is also believed that 30% of the maximum possible reduction in mortality at this age will have occurred by ten years' time.

Using an appropriate reduction factor, calculate the projected mortality rate for lives aged 60 in 20 years' time.

(6 marks)

QUESTION FIVE (20 MARKS)

An investigation took place into the mortality of residents of a care home. The investigation began on 1 January 2017 and ended on 1 January 2018. The table below gives the data collected in this investigation for 8 lives.

Date of birth	Date of entry into observation	Date of exit from observation	Whether or not exit was due to death (1) or other reason (0)
1 April 1946	1 January 2017	1 January 2018	0
1 October 1946	1 January 2017	1 January 2018	0
1 November 1946	1 March 2017	1 September 2017	1
1 January 1947	1 March 2017	1 June 2017	1
1 January 1947	1 June 2017	1 September 2017	0
1 March 1947	1 September 2017	1 January 2018	0
1 June 1947	1 January 2017	1 January 2018	0
1 October 1947	1 June 2017	1 January 2018	0

The force of mortality, μ , between exact ages 70 and 71 is assumed to be constant.

a.

i.) Estimate the constant force of mortality, μ , using a two-state model and the data for the 8 lives in the table.

(8 marks)

ii.) Hence or otherwise estimate q_{70} .

(2 marks)

b. Show that the maximum likelihood estimate of the constant force, μ , using a Poisson model of mortality is the same as the estimate using the two-state model.

(10 marks)