JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTURIAL SCIENCE

UNIVERSITY EXAMINATION FOR BACHELOR OF ACTUARIAL SCIENCE

APRIL 2022/2023

MAIN REGULAR

COURSE CODE: WAB 2103

COURSE TITLE: Mathematical Modelling

EXAM VENUE: LAB 17 **STREAM: B.Sc. Actuarial Science**

DATE:14/12/2022 **EXAM SESSION: 15.00-17.00PM**

TIME: 2 HOURS

Instructions:

1. **Answer ONE and any other two questions only.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE COMPULSORY (30 MARKS)

a) Explain the following giving an example in each case (8mks)
   i) Difference equations
   ii) Differential equations
   iii) Homogeneous difference equations
   iv) The order of a difference equation

b) It is known that, for certain income brackets, money deposited in an individualretirement Account (IRA) and its interest are tax-deferred. Karen Smith sets up an IRA at thebeginning of the year and deposited $3,000. Karen will deposit $2,000 at the end of each year, and theinterest rate is 6% compounded annually.
   i) Model this situation by a difference equation. (3mks)
   ii) What will be the future value of the account at Karen's retirement age of 60 if she starts the IRA at the age of 25? (5mks)

c) Solve the differential equation $y' = e^{x-y}.$ (3mks)

d) Find the interest on a 60- day loan of $1500 at 4.15% (5mks)

e) Solve the following difference equations to find x_{10}

   i) $x_{n+1} = 1.8x_n + 10, x_0 = 20$ (3mks)

   ii) $x_{n+1} - x_n = 3x_n + 4, x_0 = 2$ (3mks)

Question Two(20mks)

a) Assume you open an account that pays 4% compounded annually and deposit $2,000.You will deposit 2% more into your account than you deposited in the previous year.
i) Model this situation by a difference equation. (4mks)
ii) What is the total amount in the account after 20 years? (6mks)

b) You just won the lottery. You put your $5,000,000 in winnings into a fund that has a rate of return of 4%. Each year you use $300,000. Calculate the amount of money will you will have twenty years from now? (10mks)

Question Three(20mks)

a) A population of insects in a region will grow at a rate that is proportional to their current population. In the absence of any outside factors the population will triple in two weeks time. On any given day there is a net migration into the area of 15 insects and 16 are eaten by the local bird population and 7 die of natural causes. If there are initially 100 insects in the area will the population survive? If not, when do they die out? (10mks)

b) Find the solution to the following differential equation.

$\frac{dv}{dt}$=9.8−0.196v (10mks)

Question Four(20mks)

a) Your parents are considering a 30{year, $100000 mortgage that charges 0:5% interest each month. They want to pay $p monthly so that the mortgage (loan) is paid off after 360 payments (i.e., 30 years).

i) Set up and solve the model with the dynamical system (i.e., difference equation).
ii) Find the monthly payment $p. (10mks)

b) A 1500 gallon tank initially contains 600 gallons of water with 5 lbs of salt dissolved in it. Water enters the tank at a rate of 9 gal/hr and the water entering the tank has a salt concentration of 15(1+cos(t))15(1+cos(t)) lbs/gal. If a well-mixed solution leaves the tank at a rate of 6 gal/hr, calculate the amount of salt in the tank when it overflows (10mks)

Question Five(20mks)

A runner in the Ndakaine Marathon starts out too fast; as consequence her speed decreases throughout the race at a rate inversely proportional to the square root of time.

(a) Set up the differential equation that describes how her speed depends on time. (5mks)
(b) The runner starts out running 8 mph, and after one hour her speed has reduced to 7 mph. Find her speed as a function of time. 10mks)
(c) Find the function that expresses her distance from the starting line as a function of time. (5mks)