**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF BIOLOGICAL PHYSICAL MATHEMATICS AND ACTUARIAL SCIENCES**

**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION SCIENCE WITH IT, BACHELOR OF EDUCATION ARTS WITH IT AND BACHELOR OF EDUCATION ARTS (SPECIAL NEEDS) WITH IT**

**2ND YEAR 2ND SEMESTER 2024/2025 ACADEMIC YEAR**

**MAIN CAMPUS**

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**COURSE CODE: WAB 9210**

**COURSE TITLE: PROBABILITY AND DISTRIBUTION THEORY II**

**EXAM VENUE: AUD/LAB 2** **STREAM:BED**

**DATE:14/4/25** **EXAM SESSION: 9-11.00 AM**

**TIME: 2.00 HOURS**

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**Instructions:**

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

**QUESTION ONE (30 MARKS)**

a) Show that for a random sample of size $n$ from an infinite population that is $N\left(\mu,\sigma^2\right)$. The mean of the sample is given by $\mu_{\bar{x}} = \mu$, which is the population mean and the standard error of the sample mean is given by $\sigma_{\bar{x}} = \sigma / \sqrt{n}$. (4 Marks)

b) The heights of female students at a particular college are normally distributed with mean of 160 cm and standard deviation of 9 cm.

i) Given that 80% of these female students have a height less than $h$ cm, find the value of $h$. (2 Marks)

ii) Given that 60% of these female students have a height greater than $s$ cm, find the value of $s$. (3 Marks)

c) Obtain the generating function of the following sequence $\{0,0,0,0,1,1,1.1,.........\}$ (5 Marks)

d) Given that $X$ is a random variable with probability distribution function given as

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & elsewhere \end{cases}$$

i) Obtain the moment generating function of the above distribution. (4 Marks)

ii) Obtain the mean and variance of the random variable $X$. (5 Marks)

e) Given that $U$ is a chi-square with $m$ degrees of freedom, $V$ is a Chi-Square with $n$ degrees of freedom and $U$ and $V$ are Independent Random Variables. Then,

$F = \dfrac{U/m}{V/n}$ is a Fisher's $F$ variable with $m$ and $n$ degrees of freedom.

Obtain the probability distribution function of $F$. (7 Marks)

**QUESTION TWO (20 MARKS)**

a) The masses of boxes of oranges are normally distributed such that 30% of them are greater than 4.00 kg and 20% greater than 5 kg. Find the mean and standard deviation of the masses. (12 Marks)

b) A random variable $X$ is defined do have a Poisson distribution of the density of $X$ given by;

$$f(x) = \begin{cases} \dfrac{e^{-\lambda}\lambda^x}{x!} & x = 0,1,2,3,...... \\ 0 & elsewhere \end{cases}$$

i) Obtain the moment generating function of the above probability distribution function. (3 Marks)

ii) Obtain the mean and variance of $X$. (5 Marks)

**QUESTION THREE (20 MARKS)**

Given that $X$ is a continuous random variable the $X$ is said to have a Chi – square distribution with probability density function given by;

$$f(x)=\begin{cases}\dfrac{1}{\Gamma\left(\frac{n}{2}\right)2^{\frac{n}{2}}}x^{\frac{n}{2}-1}e^{-\frac{x}{2}} & x>0\\ 0 & otherwise\end{cases}$$

i) Find the moment generating function of the Chi – square distribution. (10 Marks)

ii) Obtain the mean and variance of the Chi – square distribution. (10 Marks)

**QUESTION FOUR (20 MARKS)**

Given that $X$ is a standard normal variable that is $X \sim N(0,1)$ and also $U$ is a Chi – square variables with $n$ degrees of freedom. Assuming that $X$ and $U$ are stochastically independent. Then a random variable defined by $T=\dfrac{X}{\sqrt{U/n}}$ is a student's ``$t$" variable.

i) Obtain the probability density function of $T$. (12 Marks)

ii) Obtain the mean and variance of $T$. (8 Marks)

**QUESTION FIVE (20 MARKS)**

a) A continuous random variable $X$ is said to follow Pareto (Type I) distribution if its probability density function is defined as;

$$f(x)=\frac{\alpha k^{\alpha}}{x^{\alpha+1}}, \quad x>k$$

Obtain the mean and variance of the above distribution. (12 Marks)

b) The random variable $X$ is an insurer's annual hurricane – related no – indent. Suppose that the density function of $X$ is;

$$f(x)=\begin{cases}\dfrac{2.2(250)^{2.2}}{x^{3.2}} & x>250\\ 0 & elsewhere\end{cases}$$

Calculate the mean, mode and median of the annual hurricane – related loss. (8 Marks)