



**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL
SCIENCES
UNIVERSITY EXAMINATION FOR DEGREE OF MASTER OF SCIENCE IN APPLIED
STATISTICS**

1st YEAR 1st SEMESTER 2024/2025 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SAS 804

COURSE TITLE: TEST OF HYPOTHESIS

EXAM VENUE:

STREAM: (MSc. In Applied Statistics)

DATE:14/4/25

EXAM SESSION: 9-12.00 NOON

TIME: 3.00 HOURS

Instructions:

Answer any three questions.

- i. Candidates are advised not to write on the question paper.
- ii. Candidates must hand in their answer booklets to the invigilator while in the examination room.
- iii. Statistical software for data analysis may be used.

QUESTION ONE

A researcher is investigating whether a new drug is more effective than an existing one. Let H_0 be the hypothesis that the new drug has the same effectiveness as the existing one, and H_1 be the hypothesis that the new drug is more effective.

The effectiveness is measured by a parameter θ . Assume the prior distributions:

- $H_0: \theta \sim N(0,1)$ (standard normal)
- $H_1: \theta \sim N(1,1)$ (shifted normal)

A sample of $n = 10$ patients is taken, and the sample mean effectiveness is $\bar{x} = 0.8$, with known variance $\sigma^2 = 1$.

- Compute the Bayes Factor BF_{10} in favor of H_1 . (10 marks)
- If the prior odds $P(H_1)/P(H_0) = 1$, what is the posterior probability of H_1 ? (10 marks)

QUESTION TWO

- Suppose X_1, X_2, \dots, X_n are i.i.d. from a normal distribution $N(\theta, \sigma^2)$, where σ^2 is known. Consider testing:

$$H_0: \theta = \theta_0 \text{ vs } H_1: \theta \neq \theta_0$$

- Derive the likelihood ratio test (LRT). (4 marks)
 - Show that the LRT statistic follows a chi-square distribution under H_0 . (3 marks)
 - Compute the critical region at a significance level α . (3 marks)
- Consider the Bayesian hypothesis test:

$$H_0: \theta = 0 \text{ vs } H_1: \theta \neq 0$$

where $\theta \sim N(0, \tau^2)$ a priori and data $X_1, \dots, X_n \sim N(\theta, \sigma^2)$. Compute the posterior probability of H_0 . (10 marks)

QUESTION THREE

- The following data represent the time (in hours) taken to develop a computer program by two groups of computer scientists

Group 1: 2.4, 1.5, 2.0, 1.9, 2.3, 2.9, 2.8, 2.6, 3.5, 4.5, 3.7, 5.5, 2.8, 6.5, 5.6

Group 2: 2.6, 1.5, 1.3, 2.4, 2.9, 3.0, 3.1, 3.9, 2.9, 3.9, 4.8

Assuming normality of data, perform LR Test to test the significance of

$$H_0: \mu_1 = \mu_2 \text{ against } H_1: \mu_1 \neq \mu_2$$

(10 marks)

- Suppose we have paired data (X_i, Y_i) and we wish to test:

$$H_0: X_i = Y_i \text{ vs } H_1: X_i \neq Y_i$$

using the Wilcoxon signed-rank test. Show how to compute the test statistic and its asymptotic distribution. (10 marks)

QUESTION FOUR

- a) To study the impact of Supracide on heart rate of a variety of slug, the slugs are kept in supracide for a week. The heart rate of slugs before (x_{1j} , heart-beat/minute) and after (x_{2j} , heart-beat/minute) keeping the slugs in supracide are given below

X_{1j}	10	8	12	9	9	7	8	9	10
X_{2j}	7	6	8	7	7	6	7	8	8

Test whether supracide is creating problem to the slugs (5 marks)

- b) Out of 100 students in a course 15 students got A grade and out of 75 students in another course 8 students got A. test whether the proportion of students getting grade A in two courses are the same. (7 marks)
- c) In a month there are 125 births in a hospital A, of which 55 are affected by jaundice, while in two consecutive months the proportion of jaundice-affected babies among total of 260 babies is 0.35. Test whether there is a significant difference in the proportion of jaundice-affected babies in two months (8 marks)

QUESTION FIVE

- a. In collecting the information, new born babies from two separate hospitals, the following information are recorded.

HOSPITAL 1

Gestation period X_{1j}	270	260	250	265	266	262
Birth-weight y_{1j} (in lb)	7.0	6.5	6.2	7.0	6.1	5.8

HOSPITAL 2

Gestation period X_{1j}	263	265	270	255	260	260
Birth-weight y_{1j} (in lb)	6.3	7.0	7.1	5.5	6.5	6.8

Test whether the influence of gestation on birth-weight of babies in the two hospitals are similar (10 marks)

- b. Consider a Bayesian hypothesis test where the parameter θ follows a prior $\theta \sim N(5,4)$. A sample of size $n = 25$ is collected, yielding a sample mean $\bar{x} = 5.5$ with known variance $\sigma^2 = 1$.
- i. Compute a 95% Bayesian credible interval for θ . (10 marks)
- ii. Compare this to the 95% frequentist confidence interval. (10 marks)