JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF MASTER OF SCIENCE

IN APPLIED STATISTICS

1ST YEAR 1ST SEMESTER 2018/2019 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: SAS 801

COURSE TITLE: PROBABILITY THEORY

EXAM VENUE: **STREAM: MSc. APPLIED STATISTICS**

DATE: **EXAM SESSION:**

TIME: 3.00 HOURS

Instructions:

1. **Answer ANY 3 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answAer booklets to the invigilator while in the examination room.**

QUESTION ONE (20 MARKS)

a) Show that $E|X+Y|^r \leq C_r E|X|^r + C_r E|Y|^r$ where $C_r = \begin{cases} 1 & r<1 \\ 2^{r-1} & r>1 \end{cases}$ (4 Marks)

b) Given the random variable $(X,Y) \sim GAMK(\alpha,\theta)$ where $0<\alpha<\infty$ and $0 \leq \theta < 1$ are parameters. Show that

i. $E(Y/x) = \theta x + (1-\theta)\alpha$

ii. $E(X/y) = \theta y + (1-\theta)\alpha$

iii. $\operatorname{var}(Y/x) = (1-\theta)[2\theta x + (1-\theta)\alpha]$

iv. $\operatorname{var}(X/y) = (1-\theta)[2\theta y + (1-\theta)\alpha]$

QUESTION TWO (20 MARKS)

a) Given that $X_n \xrightarrow{P} x$ and $Y_n \xrightarrow{P} y$. Let a be a real number, then $(a \in \Re)$. Then show that

i. $aX_n \xrightarrow{P} ax$

ii. $X_n + Y_n \xrightarrow{P} x + y$

iii. $X_n Y_n \xrightarrow{P} xy$

iv. $\frac{X_n}{Y_n} \xrightarrow{P} \frac{x}{y}$ where $\Pr(y_n = 0)$ and $\Pr(y = 0)$ (10 Marks)

b) Suppose that $A_1, \ldots\ldots\ldots, A_n$ are independent events with $\sum_n P_n = \infty$ where $P_n = P(A_n)$,

then $X_n = \frac{\sum_{i=1}^n 1_{A_i}}{\sum_{i=1}^n P_i} \to 1$ (6 Marks)

c) Show that convergence in probability implies convergence in distribution. (4 Marks)

QUESTION THREE (20 MARKS)

a) Suppose $X_1, X_2, \ldots\ldots\ldots$ are independent. Assume that

i. $\sum_{i=1}^n P|X_i| > b_n \to 0$

ii. $b_n^{-2} \sum_{i=1}^2 EX_i^2 1_{\{X_t \leq b_n\}} \to 0$ where $0 < b_n \uparrow_\infty$.

Show that $(S_n - a_n)/b_n \to 0$ is probability where $a_n = \sum_{i=1}^n EX_i 1_{\{X_t \leq b_n\}}$ (4 Marks)

b) Given that $X, X_1, X_2, \ldots\ldots$ are independent and identically distributed. Show that $S_n/n \to \mu_n$ is in probability for some μ_n if and only if $xP(|X_1| > x) \to 0$ as $x \to \infty$

(6 Marks)

c) Given that $(X,Y) \sim Beta(\theta_1, \theta_2, \theta_3)$ where θ_1, θ_2 and θ_3 are positive parameters. Then

$$E(Y/x) = \frac{\theta_2(1-x)}{\theta_2 + \theta_3}$$

$$\text{var}(Y/x) = \frac{\theta_2\theta_3(1-x)^2}{(\theta_2+\theta_3)^2(\theta_2+\theta_3+1)}$$ (6 Marks)

d) Show that if $Y \le X_n$, Y integrable, then $E(\underline{\lim} X_n) \le \underline{\lim} E(X_n)$ (4 marks)

QUESTION FOUR (20 MARKS)

a) Let $\{A_n\}$ be a sequence of arbitrary events, suppose $\sum_n^{\infty} \Pr(A_n) < \infty$. Show that the $\Pr(A_n)$ occurs infinitely is often 0, that is $\Pr(\overline{\lim} A_n) = 0$. (3 marks)

b) Let $\{B_n\}$ be a sequence of arbitrary events and $\sum_n^{\infty} \Pr(B_n) = \infty$. Show that the $\Pr(A_n)$ occurs infinitely often is $\Pr(\overline{\lim} A_n) = 0$. (6 Marks)

c) Given that $X, X_1, X_2, \ldots\ldots\ldots$ are independent and identically distributed and $E(X)$ exists. Show that $S_n/n \to E(X)$ and conversely if $S_n/n \to \mu$ which is finite then $\mu = E(X)$ (6 Marks)

d) Show that if $X_1, X_2, \ldots\ldots X_n$ are independent with $E(X_i) = 0$ and $\text{var}(X_i) < \infty$, $S_j = X_1 + \ldots\ldots\ldots\ldots + X_j$, then $\Pr\left(\max_{1\le j\le n}|S_j| \ge \in\right) = \frac{\text{var}(S_n)}{\in^2}$ (5 Marks)

QUESTION FIVE (20 MARKS)

Suppose $X_1, \ldots\ldots, X_n$ are independent random variables with mean 0 and variance σ^2. Let $S_n^2 = \sum_{j=1}^{n}$, σ_j^2 denotes the variance of the partial sum $S_n = X_1 + \ldots\ldots + X_n$. Show that if for every $\in > 0$ $1/S_n^2 \sum_{j=1}^{n} EX_j^2 1_{\{|X_j| >_{\in} S_n\}} \to 0$ then $S_n/s_n \to N(0,1)$ and if $\max_{j\le n} \sigma_j^2/S_n^2 \to 0$ and $S_n/s_n \to N(0,1)$ then $\frac{x^2(\Pr|X| > x)}{E(X^2 |_{\{|X| \le x\}})} \to 0$ as $x \to \infty$