JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION AND ACTUARIAL SCIENCE

3RD YEAR 2ND SEMESTER 2016/2017 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: SMA 304

COURSE TITLE: GROUP THEORY

EXAM VENUE: **STREAM: BED AND ACT SCIENCE Y3S2**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a) Show that if G is a group then, the identity element $e \in G$ is unique. (4marks)

b) State and prove the cancellation law. (7marks)

c) i) What is meant by the left and right coset of a group G. (2marks)

ii) Consider the multiplicative group $G = \{1, -1, i, -i\}$. Let $H = \{1, -1\}$ be its groups. Find all the right and the left coset of H under G and hence determine whether G is abelian. (5marks)

d) i) Distinguish between the order of a group and a cyclic group. (2marks)

ii) Let $G = \{0,1,2,3\}$ be a group of integers modulo4 under addition. Find the generators of the group G and hence determine whether G is cyclic. (6marks)

e) Prove that the intersection of two subgroups H and K of a group G is a subgroup. (4marks)

QUESTION TWO (20 MARKS)

f) i) Define a normal subgroup. (2marks)

ii) Let G be a group. Show that every subgroup of Abelian group is a normal subgroup. (5marks)

g) Given the permutations $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 4 & 5 & 2 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 1 & 5 \end{pmatrix}$

i) Express α and β in disjoint cycle form. (4marks)

ii) Determine β^{-1} (2marks)

iii) Find $\alpha^2\beta$ in cycle disjoint and hence determine its length. (5marks)

iv) Determine whether $\alpha^2\beta$ it's an even or odd (2marks)

QUESTION THREE (20 MARKS)

a) Draw the multiplication table for the set of $(\mathbb{Z}_7 \setminus \{0\}, \times)$ and determine whether it forms an abelian group. (10marks)

b) i) Define a subgroup of a group. (2marks)

ii) Prove that a nonempty subset H of a group G is a subgroup if and only if $ab^{-1} \in H$ for all $a, b \in H$. (5marks)

c) Let $\phi: G \rightarrow H$ be a homomorphism. Show that if $a \in G$, then $\phi(a^{-1}) = \phi(a)^{-1}$. (3marks)

QUESTION FOUR (20 MARKS)

a) i) Define the center of a group (2marks)

ii) Show that the centre $Z(G)$ of a group G is a subgroup of G. (5marks)

iii) Name two trivial subgroups. (2marks)

b) Prove that the number of elements in the symmetric group S_n is $n!$ (5marks)

c) The binary operation $*$ and composite $\circ$ on the set of real numbers are defined by $a * b = |a - b|$

$a \circ b = a$

i) Show that $*$ is commutative but not associative (3marks)

ii) Composite $\circ$ is associative but not commutative. (3marks)

QUESTION FIVE (20 MARKS)

a) i) Define the quotient group. (2marks)

ii) State and prove Lagrange's theorem. (8marks)

b) Prove that if $\varphi: G \to H$ is a homomorphism of a group and e is the identity of $G,$ then φ is a monomorphism if and only if $ker\varphi = \{e\}$. (5marks)

c) Let H be a subgroup of the group G and let $\in G$. Prove that $Ha = Hb$ if and only if $ab^{-1} \in H$. (5marks)