



**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES
UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE**

WITH IT

3rd YEAR 1st SEMESTER 2023/2024 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: WAB2309

COURSE TITLE: THEORY OF ESTIMATION

EXAM VENUE: STREAM: (BSc. Actuarial Science)

DATE: EXAM SESSION: Sep-Dec 2023

TIME: 2.00 HOURS

Instructions:

- i. Answer questions one and any other two.
- ii. Candidates are advised not to write on the question paper.
- iii. Candidates must hand in their answer booklets to the invigilator while in the examination room.

QUESTION ONE (30 marks)

- a) State and explain four important criteria of a good estimator (4 marks)
- b) Let x_1, x_2, \dots, x_n be a random sample of size n from a Poisson population with parameter λ . Find unbiased estimators of λ and λ^2 . Also, find the variance of the estimator of λ . (4 marks)
- c) Let x_1, x_2, \dots, x_n be a random sample of n observations selected from normal population with mean μ and variance σ^2 . Find unbiased estimators of μ and σ^2 . Also, find the variance of the estimators. (4 marks)
- d) Let x_1, x_2, \dots, x_n be a random sample of size n from $N(\mu, \sigma^2)$. Consider two estimators of μ as $T_1 = \frac{1}{n} \sum_1^n x$ and $T_2 = \frac{1}{n+1} \sum_1^n x$
Find relative efficiency of T_2 compared to T_1 (4 marks)
- e) Let x_1, x_2, \dots, x_n be a sample of n observations from a population having pdf

$$f(x) = \begin{cases} \frac{2(\theta - x)}{\theta^2}, & 0 \leq x \leq \theta \\ 0, & \text{Otherwise} \end{cases}$$

- Show that $T_n = 3\bar{x}$ is a mean square consistent estimator of θ (4 marks)
- f) Let x_1, x_2, \dots, x_n be a random sample of n observations from a population having pdf

$$f(x) = \begin{cases} \frac{1}{\theta}, & 0 \leq x \leq \theta \\ 0, & \text{Otherwise} \end{cases}$$

Show that $\left(\frac{n+1}{n}\right) x_{(n)}$ is the most efficient estimator of θ ,

Where: $x_{(n)} = \max(x_1, x_2, \dots, x_n)$ (4 marks)

- g) Let x_1, x_2, \dots, x_n be a random sample of size n observations from a population having pdf

$$f(x, \theta) = \theta e^{-\theta x}, \quad x > 0$$

The "apriori" distribution of θ is

$$f_1(\theta) = \frac{1}{\theta}, \quad 0 < \theta < \infty$$

Find Bayes estimator of θ (3 marks)

- h) Let x_1, x_2, \dots, x_n be a random sample of n observations from a population having pdf

$$f(x, \theta) = \theta e^{-\theta x}, \quad 0 < x < \infty$$

Find the estimators of θ , θ^2 and θ^3 by method of moment. (3 marks)

QUESTION TWO (20mks)

- a) Let x_1, x_2, \dots, x_n be a random sample of n observations from a population whose pdf is

$$f(x) = \frac{1}{b-a}, \quad a < x < b$$

Find the estimators of a and b by method of moment. (5 marks)

- b) Let x_1, x_2, \dots, x_m be a random sample of m observations from binomial population with parameters n and p . Find the estimators of n and p by method of moment. (5 marks)

- c) Let x_1, x_2, \dots, x_n be a random sample of n observations from a population having pdf.

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}; \quad 0 \leq x < \infty; \quad \alpha > 0; \quad \beta > 0$$

Find the estimator of α and β by method of moment. (10 marks)

QUESTION THREE (20mks)

- a) The following data represent the body weight (y kg), body length (x_1 , cm) of 12 randomly selected sea fish.

x	12	20	14	25	18	16	10	18	18	20	16	12
y	0.5	0.8	0.7	2.0	1.2	0.9	0.4	0.9	1.4	1.5	0.8	0.6

Assume the linear model of y , x as $y = \beta_0 + \beta_1 x + \varepsilon$

Estimate the parameters β_0 and β_1 by method of least squares.

(20 marks)

QUESTION FOUR (20mks)

- a) Let x_1, x_2, \dots, x_n be a random sample of n observations from a Poisson population with parameter θ such that (i) $f_1(\theta) = e^{-\theta}$, $\theta > 0$ (ii) $f_1(\theta) = \frac{1}{\alpha} e^{-\theta/\alpha}$, $\theta > 0$. Using quadratic loss function find Bayes estimator of θ .

(15 marks)

- b) Let x_1, x_2, \dots, x_n be a random sample of n observations from a population with probability function

$$P(x) = \frac{1}{N}, x=1,2, \dots, N$$

Find ML estimator of N

(5 marks)

QUESTION FIVE (20mks)

- a) Let x_1, x_2, \dots, x_n be a random sample of n observations from a Poisson population with parameter λ . Find a sufficient statistic for λ

(8 marks)

- b) Let x_1, x_2, \dots, x_n be a random sample of n observations from a population having pdf.

$$f(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x}{\beta}}; 0 \leq x < \infty; \alpha > 0; \beta > 0$$

Find joint complete sufficient statistic of α and β

(12 marks)