JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (SCIENCE)

2ND YEAR 2ND SEMESTER 2023/2024

MAIN REGULAR

COURSE CODE: SPB 9220

COURSE TITLE: MATHEMATICAL METHODS FOR PHYSICS II

EXAM VENUE: **STREAM: (BED SCI)**

DATE: **EXAM SESSION:**

TIME: 2:00HRS

Instructions:

1. **Answer question 1 (Compulsory) in Section A and ANY other 2 questions in Section B.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room**

SECTION A (COMPULSORY)

Question 1 (30 marks)

(a) Express in polar form and draw in an Argand diagram the complex number $z = 3 - 4i$. (3 marks)

(b) Newton's law of cooling can be expressed as

$$\frac{dB}{dt} = \kappa(E - B)$$

where κ and E are constants. If the initial value of B is B_0, solve for B. (3 marks)

(c) If $f(x, y) = x^2 + x^3y^2 - 2y^3$,find $f_x(3,2)$and $f_y(2,3)$. (4 marks)

(d) Use the Cauchy-Riemann equations to find out whether the function is $f(z) = x^2 - y^2 + 2ixy$ analytic. (3 marks)

(e) Find the eigenvalues of $B = \begin{pmatrix} -2 & 2 \\ 2 & 1 \end{pmatrix}$. (3 marks)

(f) Use Cramer's rule to solve for x and t in the Lorentz equations of special relativity:

$$x' = \gamma(x - vt)$$
$$f(z) = x^2 - y^2 + 2ixy$$

where $\gamma^2(1 - v^2/c^2) = 1$. (4 marks)

(g) Evaluate $(1 - i)(1 + 2i)(2 - i)$ giving your answer in the simplest form. (3 marks)

(h) It is given that $f(x, y) = 2xy^3 - 3x^2y^2 + 4y$. Determine the total differential of *f*. (3 marks)

(i) Evaluate the determinant of the matrix

$$B = \begin{pmatrix} 0 & 3 & -2 \\ -3 & 7 & -4 \\ 3 & -1 & 4 \end{pmatrix}.$$ (3 marks)

SECTION B (Attempt any TWO questions from this section)

Question 2 (20 marks)

(a) Determine the partial second order derivatives of $f(x, y) = xcosy + ye^x$, hence show that the second partial derivative is a commutative operation. (8 marks)

(b) Show that the function $f(z) = e^{-y}cosx + ie^{-y}sinx$ satisfies the Cauchy-Riemann conditions, hence find $f'(z)$. (7 marks)

(c) Find the linearization of $f(x, y) = x^2 - 2xy + \frac{1}{4}y^2 + 3$at the point (3, -2). (5 marks)

Question 3 (20 marks)

(a) For the matrix

$$A = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix},$$

find its eigenvalues and the associated eigenvectors. (10 marks)

(b) The equation of motion for a mass oscillating at the end of a spring is given by

$$m\frac{d^2y}{dt^2} = -ky$$

where *m* and *k* are constants. Solve this equation for *y* (you may take $k/m = \omega^2$). (5 marks)

(c) Determine the Laurent expansion of the function

$$g(z) = \frac{1}{z-3}$$

in the regions

(i) $|z < 3|$

(ii) $|z > 3|$ (5 marks)

Question 4 (20 marks)

(a) Find the determinant of the matrix

$$\begin{pmatrix} 4 & 3 & 0 & 1 \\ 9 & 7 & 2 & 3 \\ 4 & 0 & 2 & 1 \\ 3 & -1 & 4 & 0 \end{pmatrix}.$$ (6 marks)

(b) Solve the following system of equations using Cramer's rule.

$$-2x_1 + 3x_2 - x_3 = 1$$
$$x_1 + 2x_2 - x_3 = 4$$
$$-2x_1 - x_2 + x_3 = -3$$ (10 marks)

(c) Determine the inner product of the vectors $\vec{u}$and $\vec{v}$defined by $\vec{u} = (3+i, 1, 2-i, -5i, i+1)$ and $\vec{v} = (2i, 4-3i, 1+i, 3i, 1)$. (4 marks)

Question 5 (20 marks)

(a) Given the basis $S = \{\overrightarrow{X_1}, \overrightarrow{X_2}, \overrightarrow{X_3}\}$for $\mathbb{R}^4$where

$$\overrightarrow{X_1} = [0,0,0,7], \overrightarrow{X_2} = [2,0,0,5] \text{and } \overrightarrow{X_3} = [3,1,1,4],$$

use the Gram-Schmidt process to transform *S* to an orthonormal basis for $\mathbb{R}^4$.

(10 marks)

(b) (i) Give any two practical applications of Fourier series. (2 marks)

(ii) Find the Fourier series for the function

$$f(x) = \begin{cases} 0, -\pi \le x \le 0 \\ x, 0 \le x \le \pi \end{cases}$$ (8 marks)