



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE

ACTUARIAL

3RD YEAR 2ND SEMESTER 2023/2024

REGULAR (MAIN)

COURSE CODE: WAB 300

COURSE TITLE: FINANCIAL MATHEMATICS II

EXAM VENUE:

STREAM: (BSc Actuarial Science)

DATE:

EXAM SESSION:

TIME: 2.00 HOURS

Instructions:

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE

- a. State the assumptions underlying the Black-Scholes model.

(6 marks)

- b. A forward contract is arranged where an investor agrees to buy a share at time T for an amount K . It is proposed that the fair price for this contract at time t is:

$$f(t, S_t) = S_t - Ke^{-r(T-t)}$$

Show that this:

- (i) satisfies the boundary condition

(2 marks)

- (ii) satisfies the Black-Scholes PDE.

(5 marks)

- c. State the assumptions of mean-variance portfolio theory.

(7 marks)

- d. Investment returns (% pa), X , on a particular asset are modelled using the probability distribution

X	Probability
-7	0.04
5.5	0.96

- i.) Calculate the mean return and variance of return.

(4 marks)

- ii.) Calculate the downside semi-variance of return.

(2 marks)

- e. Investment returns (% pa), X , on a particular asset are modelled using a probability distribution with density function:

$$f(x) = 0.00075(100 - (x - 5)^2) \text{ where } -5 \leq x \leq 15$$

Calculate the shortfall probability where the benchmark return is 0% pa .

(4 marks)

QUESTION TWO

- a. Consider a portfolio consisting of equal holdings of two securities, S_x and S_y , where:
- the return on S_x is equally likely to be 5% or 10% *pa*
 - the return on S_y is equally likely to be 10% or 20% *pa*.
- (i) Calculate the means and variances of returns on each individual security. (10 marks)
- (ii) Calculate the mean and variance of the return on the portfolio as a whole, given that the correlation coefficient of the two securities is:
- 1
 - 0
- (10 marks)

QUESTION THREE

- a. Consider Security A, which has a standard deviation of investment returns of 4%. If:
- the standard deviation of the market return is 5%
 - the correlation between A's return and that of the market is 0.75
 - the risk-free rate is 5%
 - and the expected return on the market is 10%
- then calculate:
- (i) the beta of Security A (5 marks)
- (ii) Security A's expected return. (5 marks)

- b. An investor has the choice of the following assets that earn rates of return as follows in each of the four possible states of the world:

State	Probability	Asset 1	Asset 2	Asset 3
1	0.2	5%	5%	6%
2	0.3	5%	12%	5%
3	0.1	5%	3%	4%
4	0.4	5%	1%	7%
Market capitalization		10,000	17,546	82,454

Determine the market price of risk assuming CAPM holds.
Define all terms used.

(10 marks)

QUESTION FOUR

- a. Consider an investment whose returns follow a continuous uniform distribution over the range 0% to 10% *pa*.
- (i) Write down the probability density function for the investment returns. (2 marks)
 - (ii) Calculate the mean investment return. (2 marks)
 - (iii) Calculate the variance and semi-variance measures of investment risk. (2 marks)
 - (iv) Calculate the shortfall probability and the expected shortfall based on a benchmark level of 3% *pa*. (4 marks)
- b. Adam, Barbara and Charlie are all offered the choice of investing their entire portfolio in either a risk-free asset or a risky asset. The risk-free asset offers a return of 0% *pa*, whereas the returns on the risky asset are uniformly distributed over the range -5% to +10% *pa*. Assuming that each individual makes their investment choice in order to minimise their expected shortfall, and that they have benchmark returns of -2%, 0% and +2% *pa* respectively, who will choose which investment? Comment briefly on your answer. (10 marks)

QUESTION FIVE

- a. Derive the Black-Scholes equation. (20 marks)