



**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE
AND TECHNOLOGY**

UNIVERSITY EXAMINATION 2012/2013

**1ST YEAR 2ND SEMESTER EXAMINATION FOR THE DEGREE
OF BACHELOR OF EDUCATION ARTS WITH IT
(SCHOOL BASED)**

COURSE CODE: SMA 102

TITLE: CALCULUS I

DATE: 1/5/2013

TIME: 3.30-5.30PM

DURATION: 2 HOURS

INSTRUCTIONS

1. This paper contains FIVE (5) questions
2. Answer question 1 (Compulsory) and **ANY** other 2 Questions
3. Write all answers in the booklet provided

QUESTIONS 1: [30 MARKS](COMPULSORY)

(a) Evaluate the limits of the following:

(i) $\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x + 3}$ (3 marks)

(ii) $\lim_{x \rightarrow 0} \frac{(x+1)^{1/2} - 1}{x}$ (3 marks)

(b) (i) Define the term continuity of a function $f(x)$ at the point $x = a$ (3 marks)

(ii) Discuss continuity of the function $f(x)$ at the given intervals when (4 marks)

$$f(x) = \begin{cases} 5 - x, & -1 \leq x \leq 2 \\ x^2 - 1, & 2 < x \leq 3. \end{cases}$$

(c) Find the derivatives of the given functions below from first principles.

(i) $f(x) = x^2 + 2x$ (4 marks)

(ii) $y = \sin x$ (4 marks)

(d) Determine the equation of the tangent to the curve $y = x^3 - 2x^2 - 2x$ at the point (3,0). (3 marks)

(e) Differentiate the function $y = (3x - 2x^2 + x^3)^6$ (6marks)

QUESTION 2: [20 MARKS]

(a) Given the function defined as $xy + x - 2y = 1$, find its derivative. (5 marks)

(b) Differentiate the following functions with respect to x ;

(i) $y = \cos x(1 - \sin x)$. (4 marks)

$$(ii) y = \ln \frac{(2x^2 - x + 2)}{(x^2 - x)} \quad (4 \text{ marks})$$

$$(iii) y = e^{-(x^2+1)} \quad (4 \text{ marks})$$

$$(c) \text{ Show that the derivative of } \sin^{-1} x \text{ is } \frac{1}{\sqrt{1-x^2}} \quad (3 \text{ marks})$$

QUESTION 3 :[20 MARKS]

(a) Consider the curve $f(x) = 5 + 24x - 9x^2 - 2x^3$

(i) Find the y - intercept (3 marks)

(ii) Determine $f'(x)$ and $f''(x)$. (2 marks)

(iii) Find the stationary points of the curve, hence distinguish between them. (5 marks)

(b) Given the equation $x^3 + 2xy - y^2 + x = 2$,

Find

(i) the slope of the curve at $(-4, 1)$ and (3 marks)

(ii) the equation of the normal line to the curve at $(-4, 1)$. (2 marks)

(c) A curve is defined parametrically by $x = 2 \cos q$ and $y = 2 \sin q$, find the slope of the curve when $q = \frac{\pi}{3}$. (5 marks)

QUESTION 4: [20 MARKS]

(a) (i) Evaluate $\int \frac{x^2 + x + 1}{\sqrt{x}} dx$ (4 marks)

(ii) Evaluate $\int \sin \frac{1}{2} x dx$. (3 marks)

(b) (i) Show that

$$\frac{d}{dx} \left(\frac{1}{2} x - \frac{1}{4} \sin 2x \right) = \sin^2 x. \quad (3 \text{ marks})$$

(ii) Differentiate the function $\frac{x}{\sqrt{1+x^2}}$ with respect to x (4 marks)

(c) A container in the shape of a right circular cone of height 10cm and base radius of 1cm is used in catching the drop from a tap leaking at the rate of $0.1\text{cm}^3/\text{s}$. Find the rate at which the surface area of the water is increasing when the water is halfway the cone.
(6 marks)

QUESTION 5:[20 MARKS]

(a) (i) Given that $\frac{dA}{dx} = \frac{(3x+1)(x^2-1)}{x^5}$; find A in terms of x . What is the value of A when $x = 2$, if $A = 0$ when $x = 1$. (5 marks)

(ii) A curve which passes through the point $(2,0)$ has a gradient function $\frac{3x^2-1}{x^2}$. Find its equation. (3 marks)

(b) A two percent error is made in measuring the radius of a sphere. Find the percentage error in the surface area. (4 marks)

(c) A particle moves along a straight line OA with a velocity of $(6-2t)\text{m/s}$. When $t = 1$, the particle is at O .

(i) Find an expression for its distance from O in terms of t . (3 marks)

(ii) Find the time at which it is momentarily at rest and hence calculate the actual distance through which it moves during the same time interval.

(5 marks)

END