JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITYDRAFT EXAMINATION FOR BSc/BEd IN MATHEMATICS

4th YEAR 1st SEMESTER 2017/2018 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: SMA405
COURSE TITLE: PARTIAL DIFFERENTIAL EQUATIONS I

EXAM VENUE: **STREAM: BSc Y4S1**

TIME: 2 HOURS **EXAM SESSION:**

Instructions:

Answer question1 and any other two questions

1. **Show all the necessary working**
2. **Candidates are advised not to write on the question paper**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room**

QUESTION ONE(30 MARKS) -COMPULSORY

(a) Explain the three main properties of classifying the partial differential equation

$$a\frac{\partial^2 u}{\partial x^2}+b\frac{\partial^2 u}{\partial x\partial y}+c\frac{\partial^2 u}{\partial y^2}+d\frac{\partial u}{\partial x}+e\frac{\partial u}{\partial y}+fu+g=0 \qquad (3\text{Mks})$$

(b) If the points $p(a,b)$ of the curve $g(x,y)=4x^2y-y^2-8x^2-2x^4+1400$

satisfy system of the equations $\frac{\partial g}{\partial x}=0$, $\frac{\partial g}{\partial y}=0$

discuss the nature of such point $p(a,b)$ (8Mks)

(c) Prove that the Pfaffian differential equation

$(y^2+yz)dx+(xz+z^2)dy+(y^2-xy)dz=0$ is integrable (8Mks)

(d) Categorize the given partial differential equations below;

(i) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+xy=0$ (ii) $\frac{\partial u}{\partial t}=1002\frac{\partial^2 u}{\partial x^2}$ (iii) $\frac{\partial^2 u}{\partial x^2}+y^3\frac{\partial^2 u}{\partial y^2}=0$ (iv)

$\frac{\partial u}{\partial t}+t\frac{\partial^2 u}{\partial t\partial x}=\frac{\partial^2 u}{\partial x^2}$ [5 marks]

(e) State degree of the partial differential equations below;

(i) $\frac{\partial^2 u}{\partial x^2}+\sqrt{\frac{\partial^2 u}{\partial y^2}}+2y^4=0$ (ii) $\left(\frac{\partial u}{\partial t}\right)^9=1002\frac{\partial u}{\partial x}$ (iii) $\frac{\partial^2 u}{\partial x^2}+y^3\frac{\partial^2 u}{\partial y^2}=0$

and identify which of the equations are linear [6 marks]

QUESTION TWO (20Mks)

Show that the differential equation $(x^2z-y^3)dx+3xy^2dy+x^3dz=0$ is

i) homogenous
ii) integrable
iii) in Pfaffian form

and solve it. **(20Mks)**

QUESTION THREE (20 Mks)

Given the curve $F(x,y,z)=8x^2+24y^2+16z^2+24x+16z-10$

i)evaluate $\frac{\partial F}{\partial x}=0$, $\frac{\partial F}{\partial y}=0$ $\frac{\partial F}{\partial z}=0$

ii)determine and classify all the critical points

iii) construct H_F the Hermitian matrix F

iv)obtain the minimum and maximum values of F

QUESTION FOUR (20 Mks)

a) Use Charpit's method to solve partial differential equation; $p^2-3q^2=z$. (6 marks)

b) Consider a perfectly flexible elastic string, stretched between two points at $x=0$ and $x=1$ with uniform tension τ.
If the string is displaced slightly from its initial position while the ends remain fixed , and then released , the string will oscillate . The position u in the string at any instant will then be a function of its distance from one end x, of the string and also of time t $u=u(x,t)$,

i) Show that the equation of the motion is given by the partial differential equation

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial t^2}$$

ii) Use variable separation, to solve the equation $\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial t^2}$ given the boundary conditions
$u(0,t)=u(1,t)=0$ for all time $t\geq 0$
and the initial condition
$u(x,0)=\sin 2\pi$, $u_t(x,0)=0$ (14 marks)

QUESTION FIVE (20Mks)

Use the Jacobi method to solve the partial differential equation $p^2x+q^2x=2z$
(20 mark)