JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

3RD YEAR 1ST SEMESTER 2015/2016 ACADEMIC YEAR

REGULAR

COURSE CODE: SAC 301

COURSE TITLE: METHODS OF ACTUARIAL INVESTIGATIONS MATHEMATICS I

EXAM VENUE: LR 7 **STREAM: BSc. Actuarial Sc.**

DATE: 25/04/16 **EXAM SESSION: 2.00 – 4.00 PM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE

(a) Define the following terms as used in Financial Mathematics.

i. Immunisation.

ii. Macauly duration.

iii. Continuous rate.

iv. Certificate of deposit.

v. Ex-dividend share. [5 marks]

(b) Describe the characteristics of Government Bills. [3 marks]

(c) A ten-year bond with half yearly coupons of 6% pa has just been issued with a redemption yield of 9% pa effective. It is redeemable at par. What price would an investor paying 15% tax on income pay for the bond? Tax payments are due four months after each coupon is received. [5 marks]

(d) Calculate, using 10% pa interest, the convexity of the following assets, each of which has a discounted mean term of 11 years. Comment on your answers.

i. Asset A is an 11-year zero coupon bond.

ii. Asset B will provide a lump sum payment of 9,663 in 5 years time and a lump sum payment of 26,910 in 20 years time.

iii. Asset C is a level annuity of 1 pa payable annually in arrears for 50 years. [9 marks]

(e) If the n year spot rates can be approximated by the function $0.06-0.03e^{-0.1n}$, calculate the one-year forward rate at time 10. [2 marks]

(f) A fund must make payments of 50,000 at the end of the sixth and eighth years. Show that, if interest rates are currently 7% pa at all durations, immunisation to small changes in interest rates can be achieved by holding an appropriately chosen combination of a 5- year zero-coupon bond and a 10-year zero-coupon bond. [6 marks]

QUESTION TWO

(a) What happens to yields of fixed interest securities if:

i. Bond prices fall

ii. Demand for fixed-interest securities falls

iii. The government issues many more stocks

iv. Institutional investors suddenly decide to invest less in equities and more in fixed-interest securities

v. Bond prices rise. [10 marks]

(b) The current annual term structure of interest rates is: (6%; 6%; 6%; 6%; 7%) Calculate the gross redemption yield of a five-year fixed-interest security redeemable at par if the annual coupon is

i. 2%. [2 marks]

ii. 4%. [2 marks]

(c) Evaluate the discounted mean term of a bond redeemable at par in 10 years time with annual coupons of 8% at interest rates of 5%, 10% and 15%. Hence sketch a graph of the discounted mean term as a function of the interest rate over the range 5% to 15%. [6 marks]

QUESTION THREE

(a) In a particular bond market, the two-year par yield at time t = 0 is 4.15% and the issue price at time t = 0 of a two-year fixed interest stock, paying coupons of 8% annually in arrears and redeemed at 98, is 105.40 per 100 nominal. Calculate:

i. the one-year spot rate

ii. the two-year spot rate. [6 marks]

(b) An insurance company has liabilities consisting of 11annual payments of 1 million with the first payment due to be made in 10 year's time and the last payment due to be made in 20 year's time. The rate of interest is 6% per annum effective.

i. Show that the discounted mean term of these liabilities is 14.42 years. [5 marks]

ii. The insurance company holds two zero coupon bonds, one paying X in ten year's time and the other paying Y in 20 year's time. Find the values of X and Y such that Redington's first two conditions for immunisation from small changes in interest rate. [7 marks]

iii. Explain, without making any further calculations whether you would expect Redington's third condition for immunisation to be satisfied for the values of X and Y calculated for in (ii). [2 marks]

QUESTION FOUR

(a) An economist's model of interest rate indicates that the n-yr spot rate of interest is $0.1(1 + e^{-0.1n})^{-1}$ According to this model, what is the price of a 10 year zero coupon bond redeemable at par?. [3 marks]

(b) An investor purchases a bond 3 months after issue. The bond will be redeemed at par ten years after issue and pays coupons of 6% per annum annually in arrears. The investor pays tax of 25% on both income and capital gains (with no relief for indexation). Calculate the purchase price of the bond per 100 nominal to provide the investor with a rate of return of 8% per annum effective. [6 marks]

(c) The one-year forward rates for transactions beginning at times $t = 0$, 1, 2 are f_t where

$$f_0 = 0.06, \qquad , f_1 = 0.065 \qquad f_2 = 0.07$$

Calculate the par yield for a 3-year bond. [3 marks]

(d) For the last 10 years a man has paid 50 at the start of each month into a savings account that has achieved a real rate of interest of 3% per annum over this period. If inflation has been at a constant rate of 5% per annum, calculate the balance of the man's account today. [4 marks]

(e) A fixed interest stock with a coupon of 8% per annum payable half yearly in arrears can be redeemed at the option of the lender (i.e the investor) at any time between 10 and 15 years from the date of issue. What price should an investor subject to tax at 25% on income, who wishes to obtain a net yield of at least 7% per annum, pay for 100 nominal of this stock? [4 marks]

QUESTION FIVE

(a) A loan of nominal amount of 100000 is to be issued bearing coupons payable quarterly in arrears at a rate of 7% per annum effective. Capital is to be redeemed at 108% on a coupon date between 15 and 20 year's inclusive. The date of redemption is a the option of the borrower. An investor who is liable to income at 25% and capital gains tax at 35% wishes to purchase the entire loan at the date of issue. Calculate the price which the investor should pay to ensure that a net effective yield of atleast 5% per annum. [9 marks]

(b) Explain how expectations theory can be modified by both liquidity preference and market segmentation theories. [6 marks]

(c) An investor, who is liable to income tax at 20% but is not liable to capital gains tax, wishes to earn a net effective rate of return of 5% per annum. A bond bearing coupons payable half-yearly in arrear at a rate 6.25% per annum is available. The bond will be redeemed at par on a coupon date between 10 and 15 years after the date of issue, inclusive. The date of redemption is at the option of the borrower. Calculate the maximum price that the investor is willing to pay for the bond. [5 marks]