JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

SPECIAL RESITS EXAMINATIONS

MAIN CAMPUS

COURSE CODE: SMA 301

COURSE TITLE: ODE

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (COMPULSORY) (30 marks)

a) Determine:
   i) the order,
   ii) the degree,
   iii) the unknown function, and
   iv) the independent variable

   for differential equation

   $(y''')^2 + 2y^4 (y'')^5 + 5y^8 = e^x$ (4 marks)

b) Find a solution to the boundary-value problem $y'' + 4y = 0;\ y\left(\frac{\pi}{8}\right) = 0,\ y\left(\frac{\pi}{6}\right) = 1.$ If the general solution to the differential equation is $y(x) = C_1 \sin 2x + C_2 \cos 2x.$ (6 marks)

c) Show that the solution of the equation $\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L}$ is $i = \frac{V}{R} + Ce^{-(R/L)t}$ (4 marks)

d) Show that $y = \ln x$ is a solution of $xy'' + y' = 0$ on $\wp = (0, \infty)$ but is not a solution on $\wp = (-\infty, \infty)$ (4 marks)

e) Solve $3y'' + 2y' + y = 0$. (4 marks)

f) Show that a separable first order differential is always exact. (4 marks)

g) Assume a population $P(t)$. Suppose research has shown that its rate of growth is directly proportional to the amount present at time t. Set up the model relationship. Hence obtain its general solution. (4 marks)

QUESTION TWO (20 marks)

a) Solve the following differential equation

   $e^x dx - (1 + e^x) y dy;\quad y(0) = 1$ (6marks)

b) Solve the initial value problem:

   $y'' + y' = 3e^{t/2}\ ;\quad y(0) = 4,\quad y'(0) = 3.$

   State the largest interval in which the solution is guaranteed to uniquely exist. (7marks)

c) Solve the initial value problem

   $y'' + 2y' - 3y = 0,\ \ y(2\pi) = 1,\ y'(2\pi) = 13.$ (7 marks)

QUESTION THREE (20 marks)

a) Determine whether or not $\left(1+y^2\sin 2x\right)dx-2y\cos^2 xdy=0$ is exact. If exact, find the solution. (7marks)

b) Find the solution of the given differential equation
$\left(x\ln x\right)y'+y=2\ln x$; $y\left(e\right)=0$ (6marks)

c) Show that
$$y'=\frac{x^2+2xy-y^2}{x^2-2xy-y^2};\ y\left(1\right)=-1$$
is homogeneous and find its solution. (7 marks)

QUESTION FOUR (20 marks)

a) Solve the initial-value problem using the method of undetermined coefficients
$y''-4y=e^x\cos x,\quad y\left(0\right)=1,\quad y'\left(0\right)=2.$ (11 marks)

b) Solve the differential equation using the method of variation of parameters
$$y''-2y'+y=\frac{e^t}{t}$$ (9 marks)

QUESTION FIVE (20 marks)

a) A certain city had a population of 25000 in 1960 and a population of 30000 in 1970. Assume that its population will continue to grow exponentially at a constant rate. What populations can its city planners expect in the year 2000? (5 marks)

b) A particle moves vertically under the force of gravity against air resistance Kv^2, where K is a constant. The velocity at any time is given by the differential equation
$$\frac{dv}{dt}=g-Kv^2$$
If the particle starts off from rest, show that
$$v=\frac{\lambda\left(e^{2\lambda kt}-1\right)}{\left(e^{2\lambda kt}+1\right)}$$
Such that $\lambda=\sqrt{\frac{g}{K}}$. Then find the velocity as the time approaches infinity. (6 marks)

c) Equation $y''+9y=14\sin 4t$ describes a spring block system that is driven by an oscillatory external for $f\left(t\right)=14\sin 4t$ in the absence of friction. If the block as an initial position $y\left(0\right)=4$ and an initial velocity $y'\left(0\right)=1$. Find the solution of the initial value problem. (9 marks)